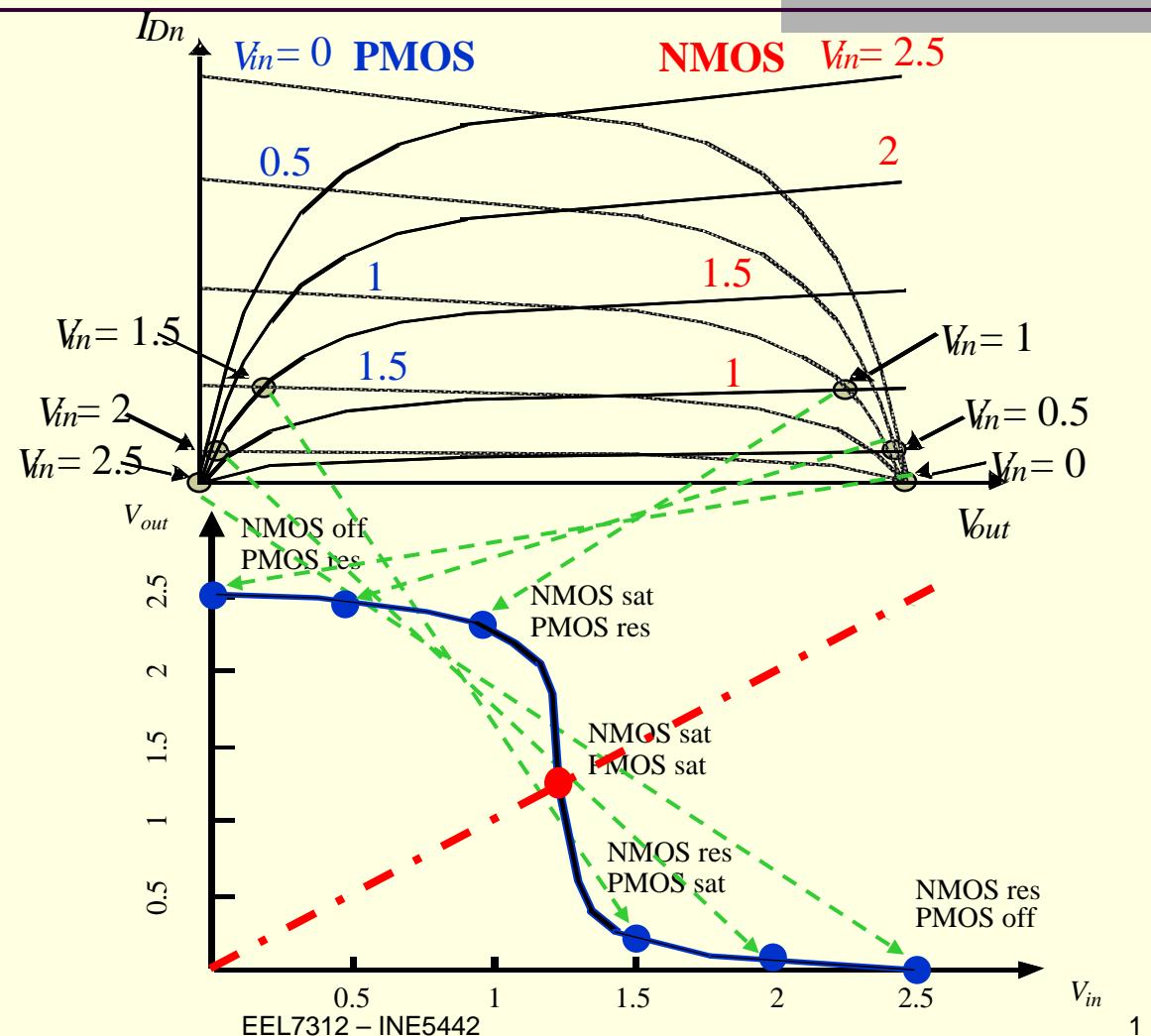
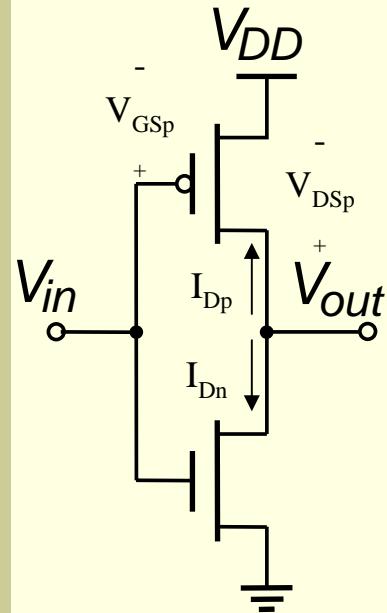


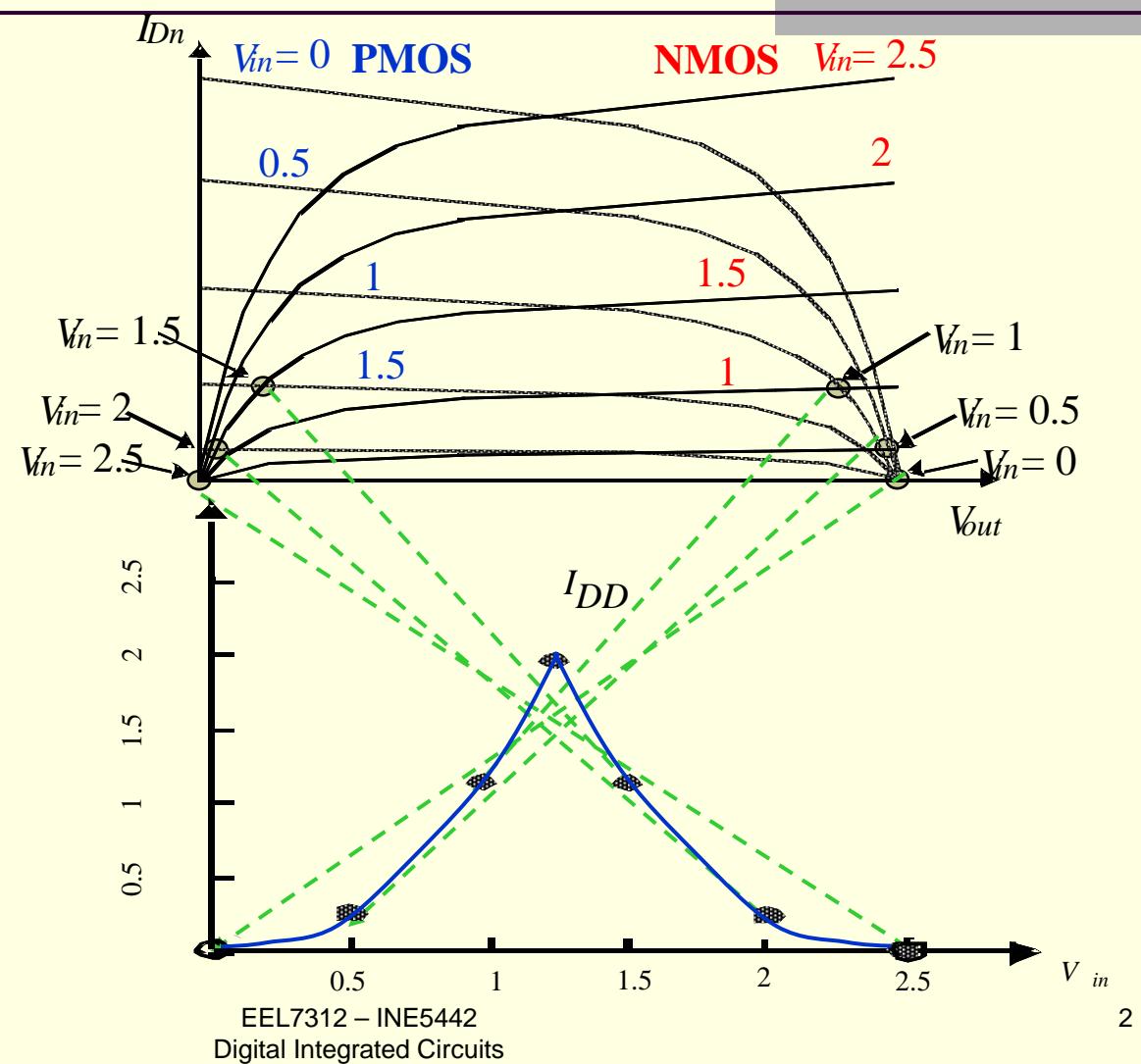
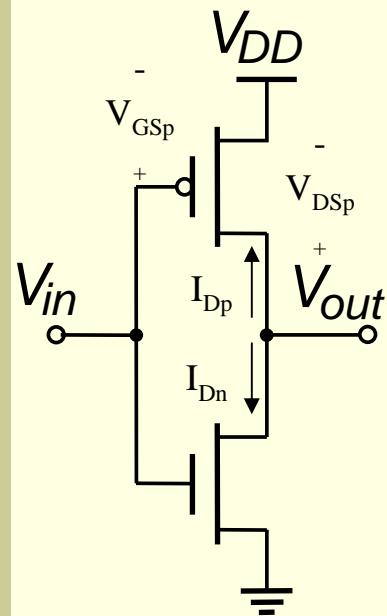
# Static characteristics - 4

VTC



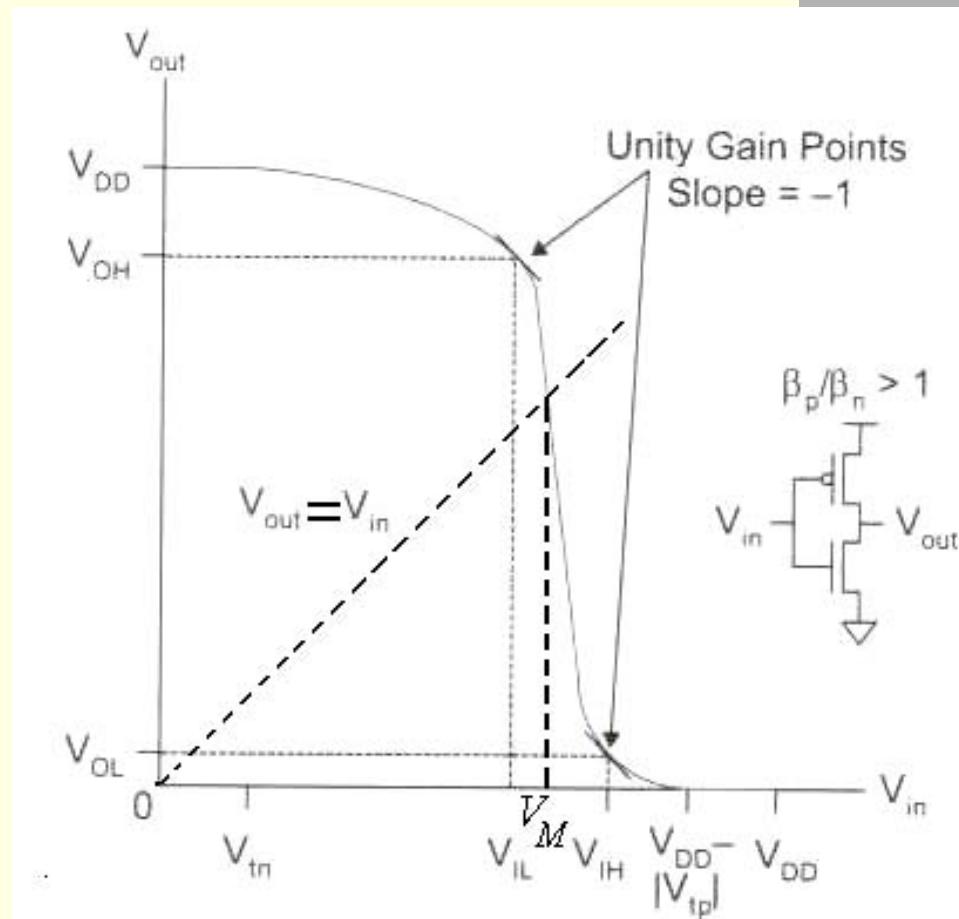
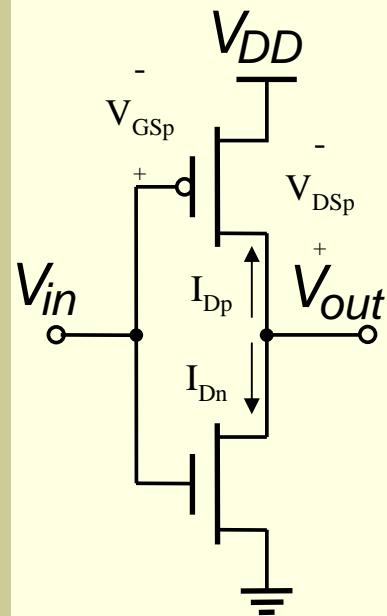
Source: Rabaey

# Static characteristics - 5 Short-circuit current



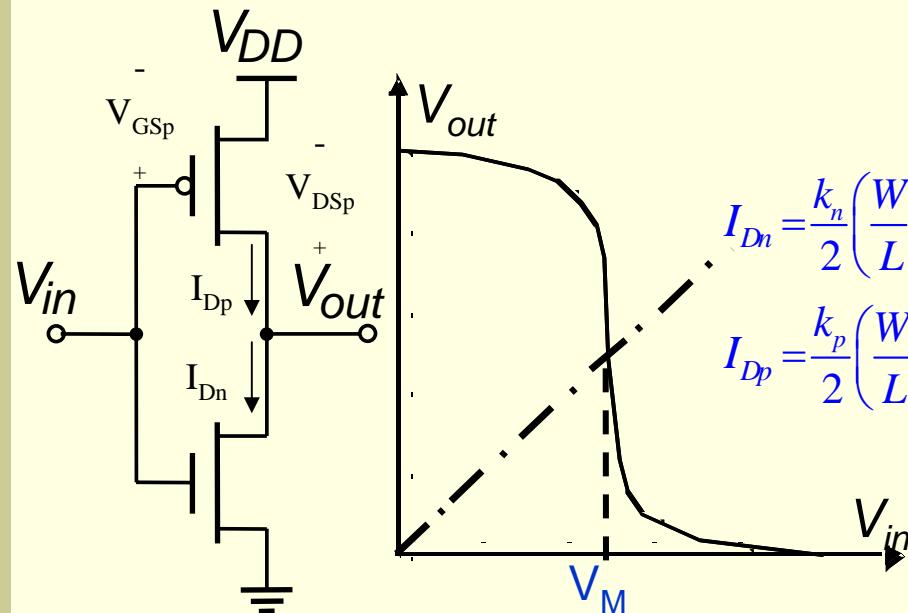
Source: Rabaey

# Static characteristics - 6 Switching threshold - 1



Source: Weste & Harris

# Static characteristics - 7 Switching threshold - 2



Experimental determination of  $V_M$ :  
short-circuit between input and output

$$I_{Dn} = \frac{k_n}{2} \left( \frac{W}{L} \right)_n (V_{GSp} - V_{Tn})^2 (1 + \lambda \frac{|V_{DSn}|}{V_{Tn}}) \approx \frac{k_n}{2} \left( \frac{W}{L} \right)_n (V_M - V_{Tn})^2$$

$$I_{Dp} = \frac{k_p}{2} \left( \frac{W}{L} \right)_p (V_{GSp} - V_{Tp})^2 (1 + \lambda \frac{|V_{DSp}|}{V_{Tp}}) \approx \frac{k_p}{2} \left( \frac{W}{L} \right)_p (V_{DD} - V_M - |V_{Tp}|)^2$$

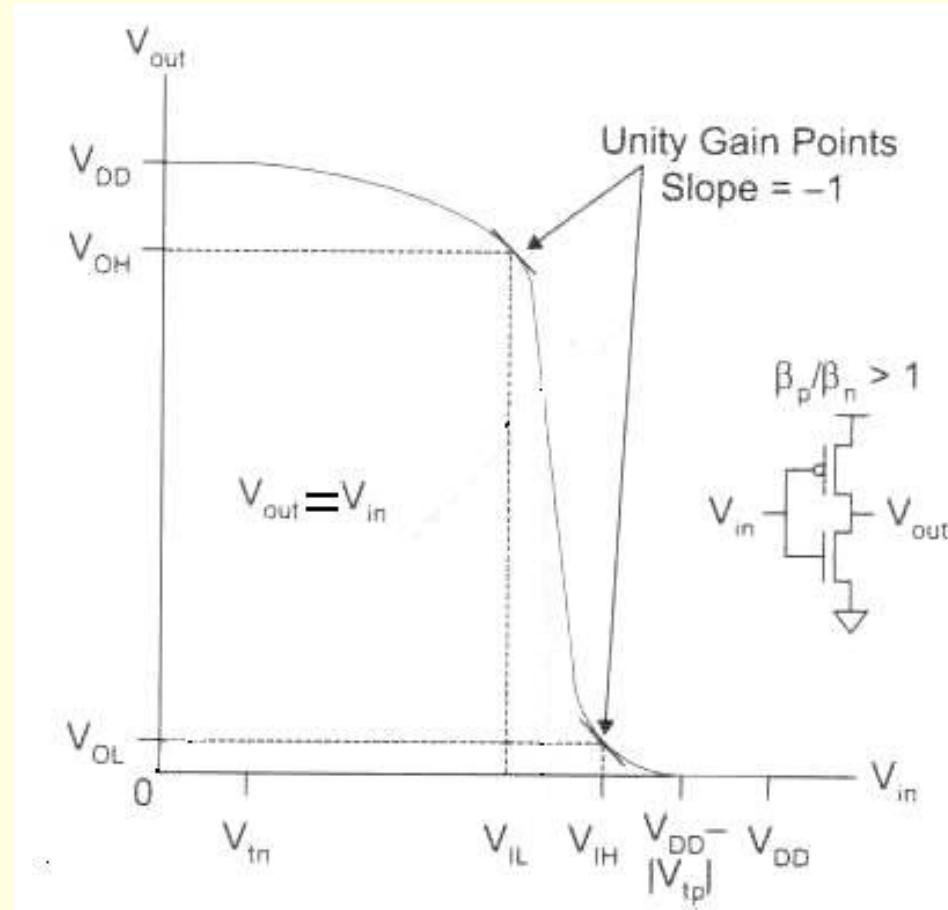
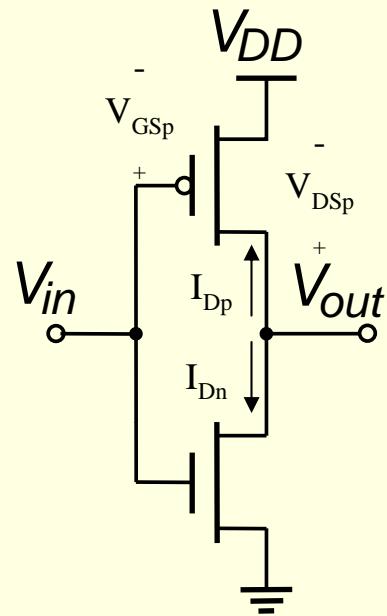
Usually  $\lambda V_{DS} \ll 1$

$$I_{Dn} = I_{Dp} \rightarrow V_M = \frac{V_{Tn} + rV_{Tp}}{1+r} + \frac{rV_{DD}}{1+r},$$

$$r = \sqrt{\frac{k_p}{k_n} \frac{(W/L)_p}{(W/L)_n}}$$

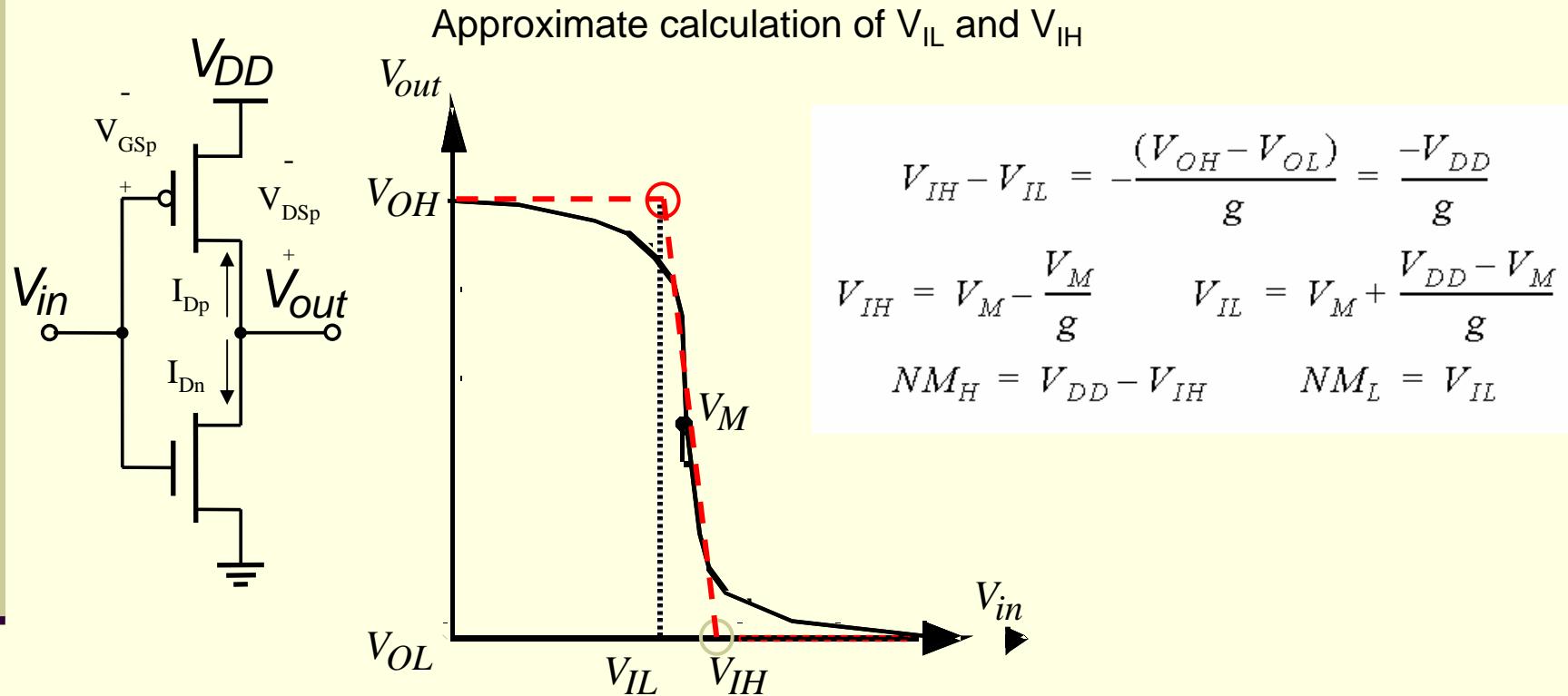
Example:  $V_{DD}=2.5$  V,  $V_{Tp}=-0.4$  V,  $V_{Tn}=0.43$  V. What is  $V_M$  for  $r= 0.5, 1.0$ , and  $1.5$ ? Answer:  $V_M=0.98, 1.26$ , and  $1.43$  V, respectively.

# Static characteristics - 8 Noise margins - 1



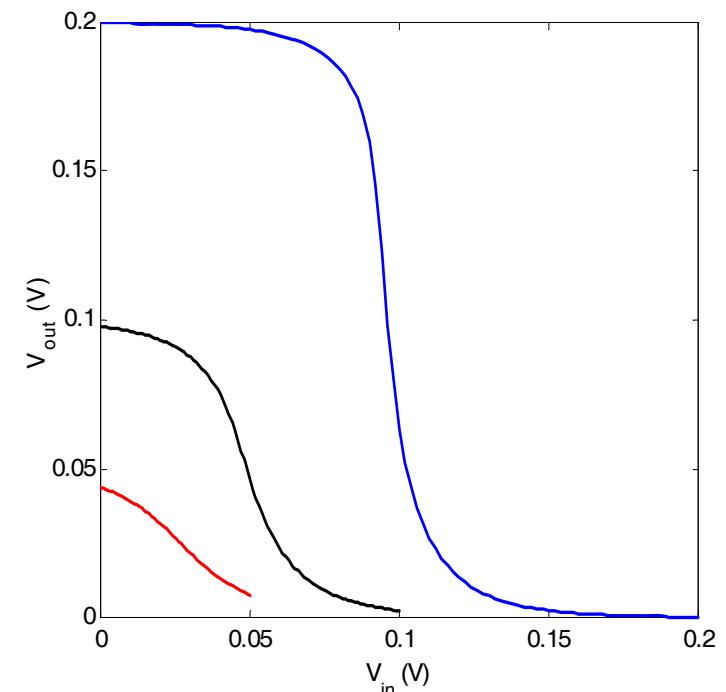
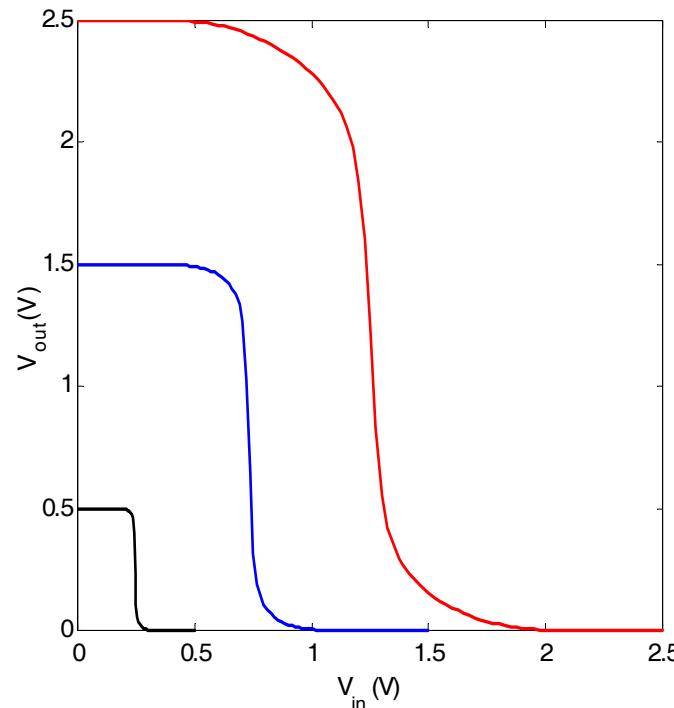
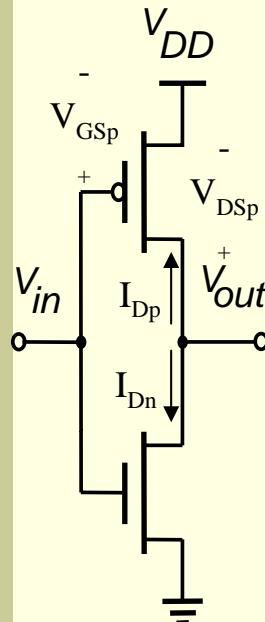
Source: Weste & Harris

# Static characteristics - 9      Noise margins - 2



For regeneration  $-g > 1$ ,  $g$  is the gain in transition region

# Static characteristics - 10 Scaling the supply voltage

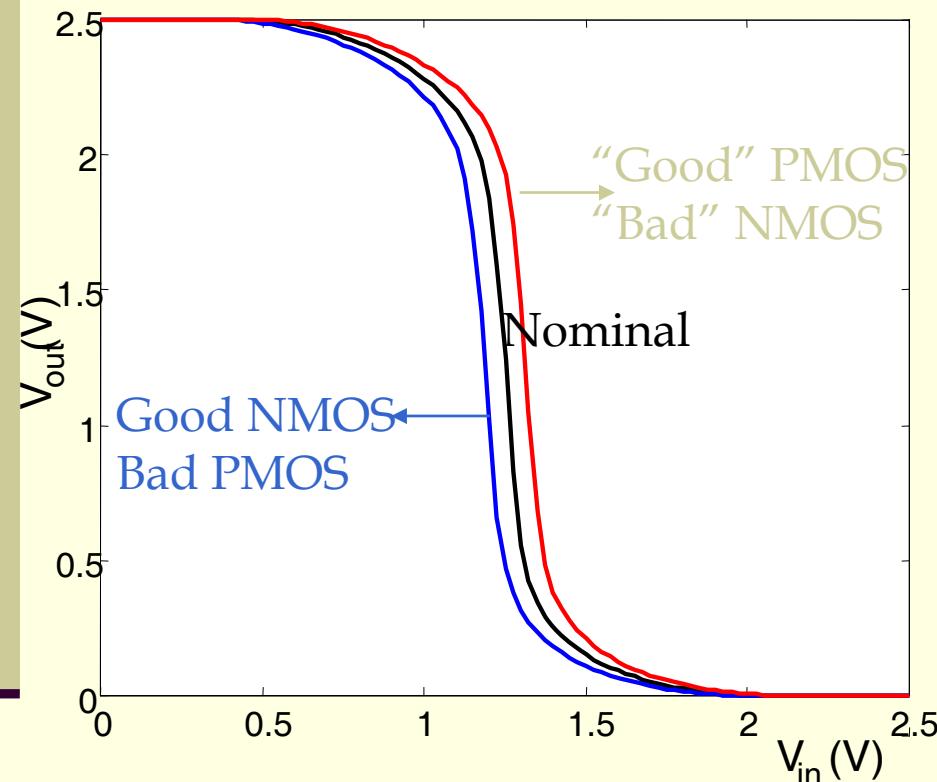


Effects of supply voltage reduction:

- Energy dissipation decreases but gate delay increases
- dc characteristic becomes more sensitive to variations in device parameters
- Signal swing reduces making the design more sensitive to external noise sources that do not scale

Source: Rabaey

# Static characteristics -11 Impact of Process Variations



Notes:

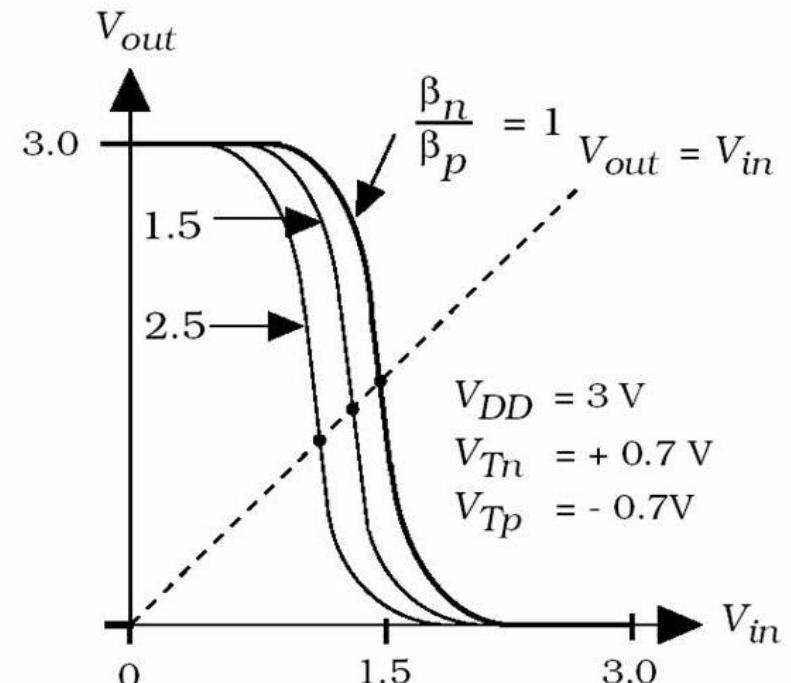
1.  $k'_n \approx 2$  to  $3 k'_p$
2. For  $\beta_n = \beta_p$  and  $V_{Tn} = -V_{Tp}$ ,  $V_M = V_{DD}/2$

Source: Rabaey

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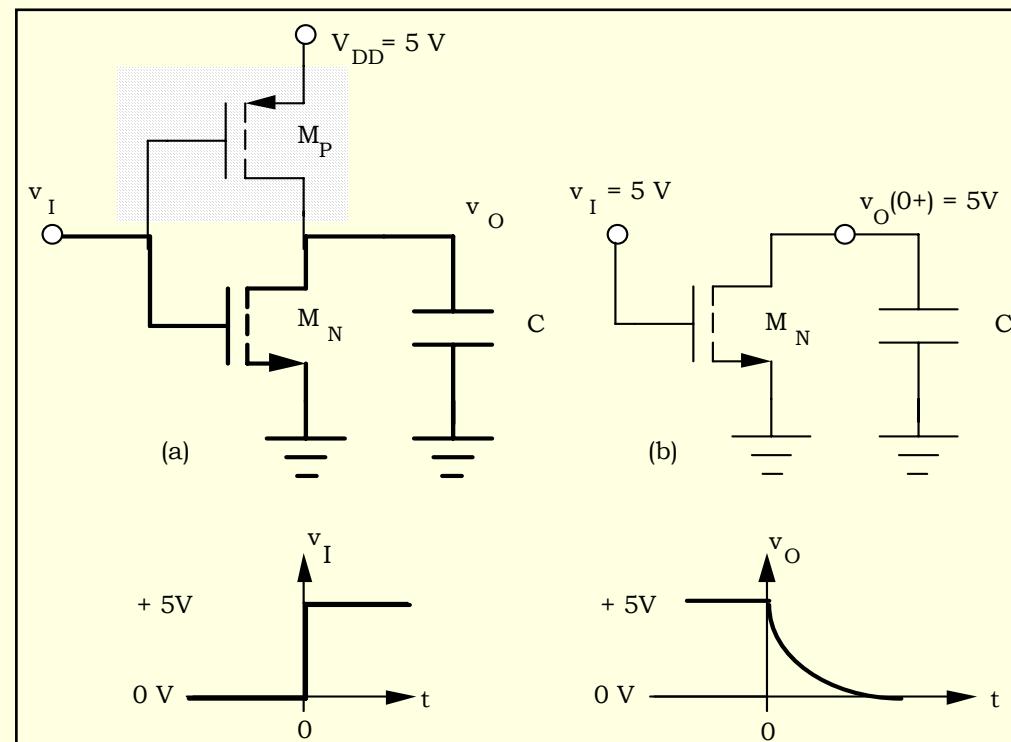
$$\beta = k' \frac{W}{L}$$

Source: Uyemura



# Dynamic operation - 1

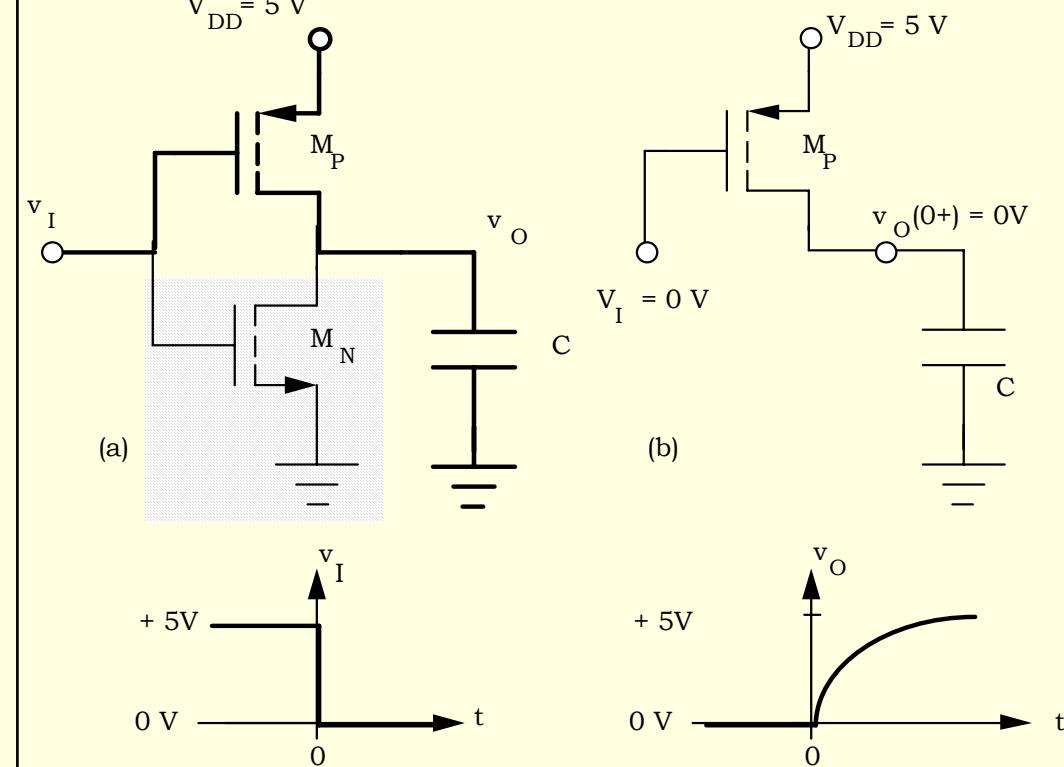
High-to-low output transition  
in a CMOS inverter



C: load capacitance + interconnect capacitance +  
capacitances associated with the inverter transistors

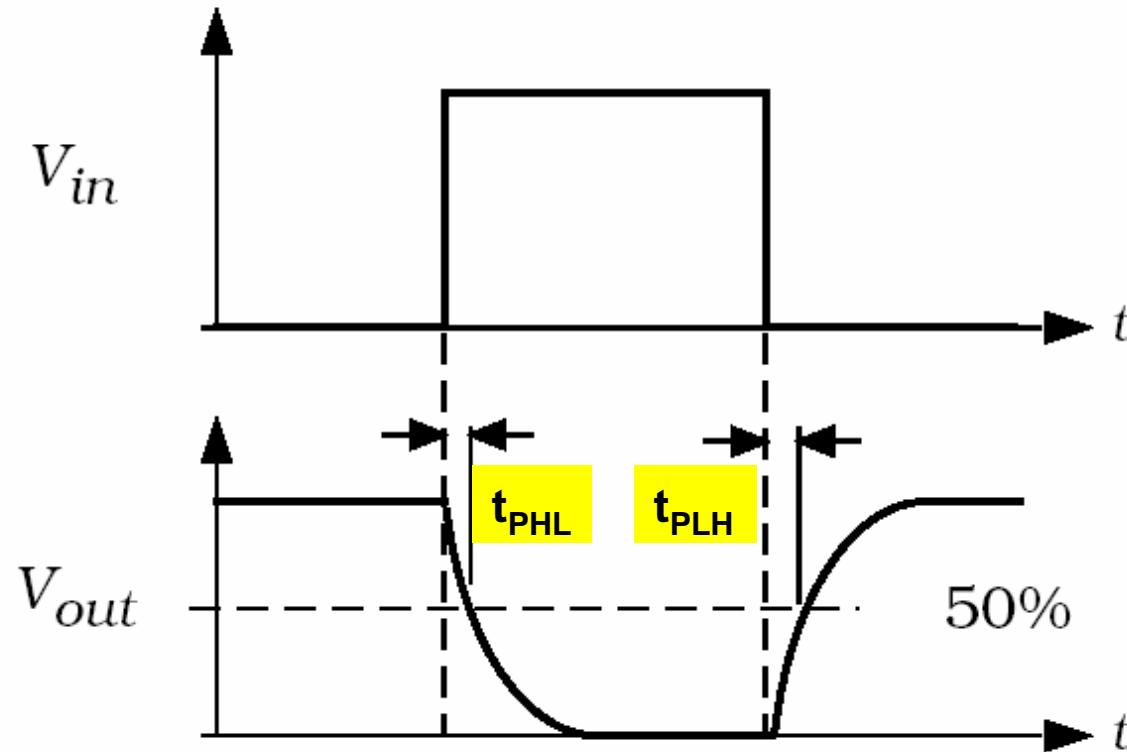
# Dynamic operation - 2

Low-to- high output  
transition in a CMOS inverter



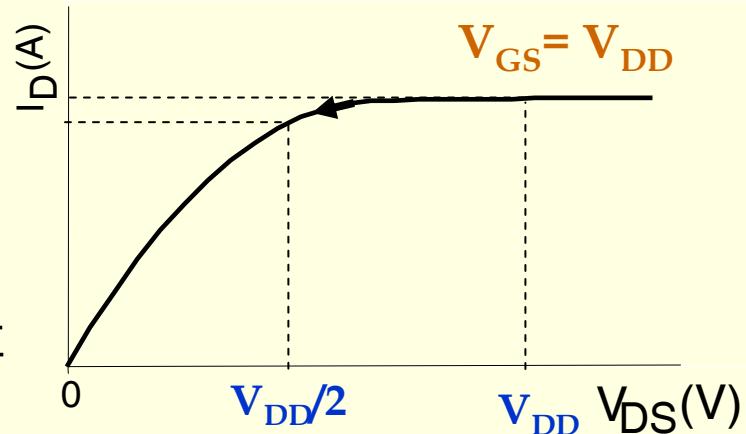
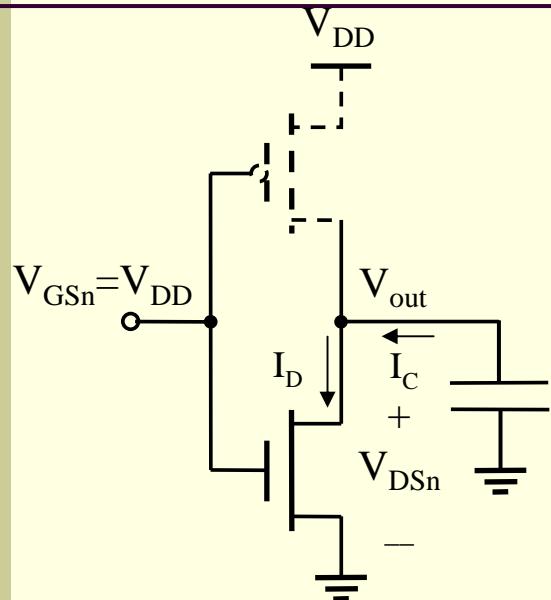
$C$ : load capacitance + interconnect capacitance +  
capacitances associated with the inverter transistors

# Dynamic operation - 3



# Dynamic operation - 4

Propagation  
delay - 1



$$I_D = I_C = -C \frac{dV_{out}}{dt}$$

$$t = 0 \rightarrow V_{out} = V_{DD}$$

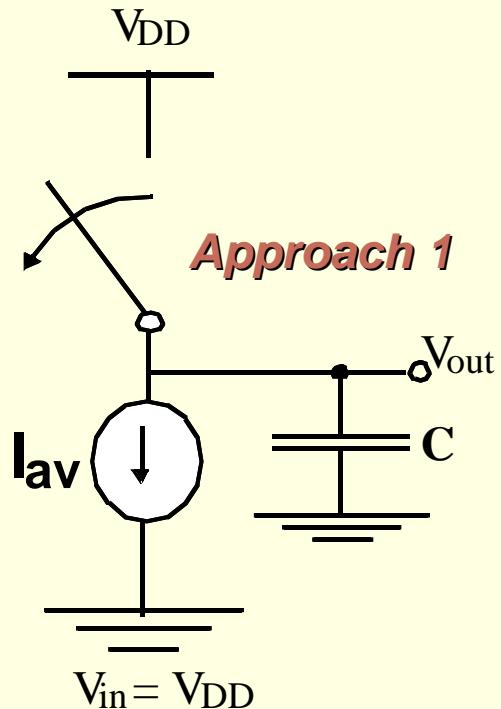
$$t = t_{PHL} \rightarrow V_{out} = V_{DD}/2$$

$$\int_0^{t_{PHL}} dt = - \int_{V_{DD}}^{V_{DD}/2} C \frac{dV_{out}}{I_D} \rightarrow t_{PHL} = \frac{CV_{DD}/2}{I_{Dav}}$$

$$I_{Dav} = \frac{1}{V_{DD}/2} \int_{V_{DD}/2}^{V_{DD}} I_D(V_{DS}) dV_{DS}$$

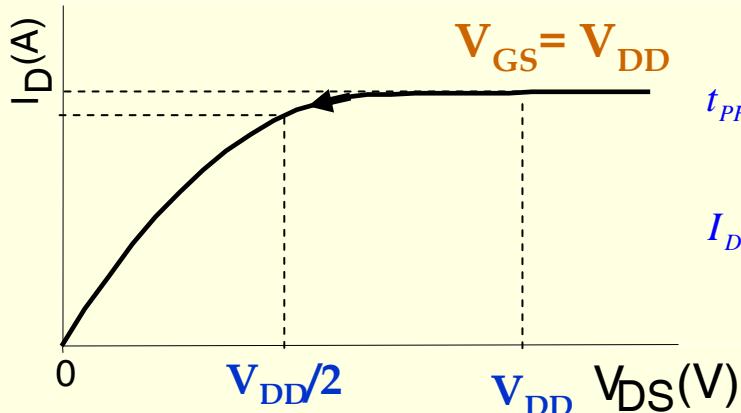
# Dynamic operation - 5

## Propagation delay - 2



Let us assume that

In this case we have



$$t_{PHL} = \frac{CV_{DD}/2}{I_{Dav}}$$

$$I_{Dav} = \frac{1}{V_{DD}/2} \int_{V_{DD}/2}^{V_{DD}} I_D(V_{DS}) dV_{DS}$$

$$I_D \approx \frac{k_n}{2} \left( \frac{W}{L} \right)_n (V_{GS} - V_{Tn})^2 \text{ for } V_{DS} > V_{GS} - V_{Tn}$$

$$I_D \approx k_n \left( \frac{W}{L} \right)_n [(V_{GS} - V_{Tn}) V_{DS} - V_{DS}^2 / 2] \text{ for } V_{DS} \leq V_{GS} - V_{Tn}$$

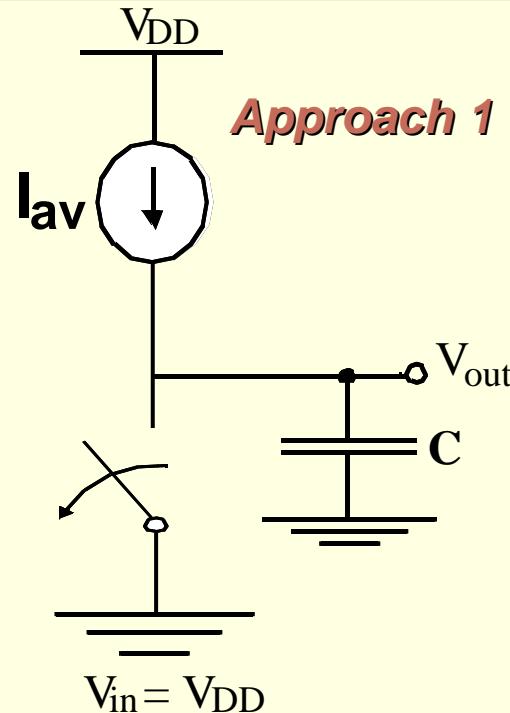
$$I_{Dav} \approx I_{Dsat} = \frac{k_n}{2} \left( \frac{W}{L} \right)_n (V_{DD} - V_{Tn})^2 \text{ and that } V_{DD} \gg V_{Tn}$$

$$t_{PHL} = \frac{CV_{DD}/2}{I_{Dav}} \approx \frac{CV_{DD}/2}{\frac{k_n}{2} \left( \frac{W}{L} \right)_n (V_{DD} - V_{Tn})^2};$$

$$t_{PHL} \approx \frac{C}{k_n \left( \frac{W}{L} \right)_n V_{DD}}$$

# Dynamic operation - 6

## Propagation delay - 3



$$I_{Dav} \cong I_{Dsat} = \frac{k_p}{2} \left( \frac{W}{L} \right)_p \left( V_{DD} + V_{Tp} \right)^2 \text{ and that } V_{DD} \gg -V_{Tp}$$

$$t_{PLH} = \frac{CV_{DD}/2}{I_{Dav}} \approx \frac{CV_{DD}/2}{\frac{k_p}{2} \left( \frac{W}{L} \right)_n \left( V_{DD} + V_{Tp} \right)^2};$$

$$t_P = \frac{t_{PLH} + t_{PHL}}{2}$$

$$t_{PLH} \approx \frac{C}{k_p \left( \frac{W}{L} \right)_p V_{DD}}$$

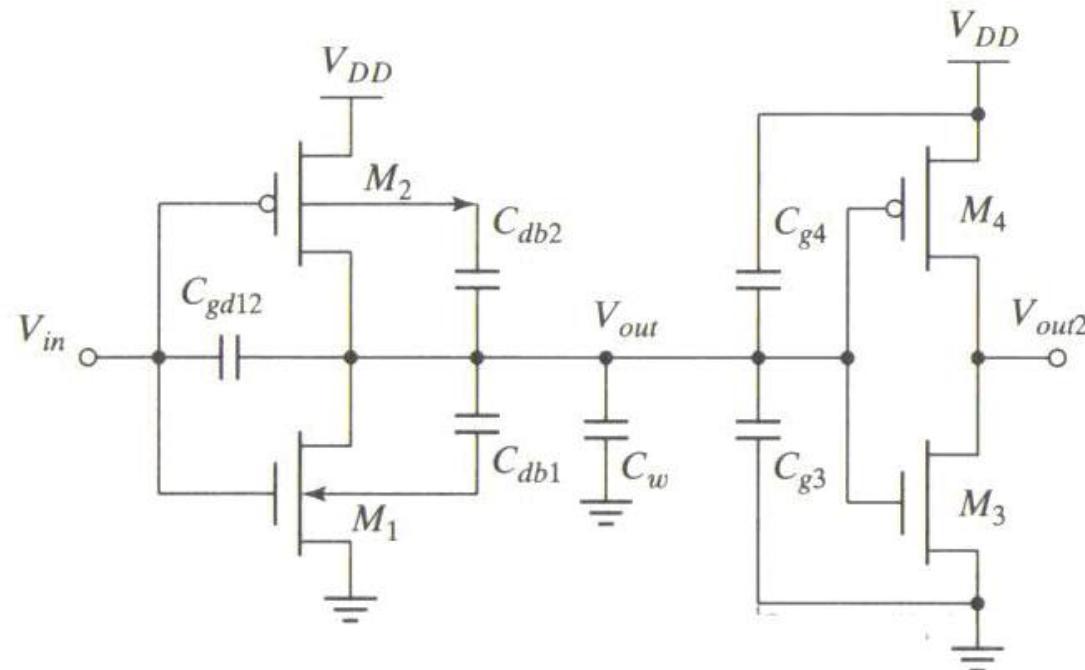
$$t_{PHL} \approx \frac{C}{k_n \left( \frac{W}{L} \right)_n V_{DD}}$$

### Comments:

- $k_n \approx 2-3 k_p$ ,  $k_{n,p} = \mu_{n,p} \cdot C_{ox}$
- Increasing  $V_{DD}$  reduces  $t_p$  but power goes up
- $t_{PLH}$  can be  $\approx t_{PHL}$  by making  $(W/L)_p \approx 2-3(W/L)_n$   
**BUT C is dependent on transistor dimensions**
- C includes load (fan-out), wire, inverter “self-capacitance”
- C is non linear

# Dynamic operation - 7

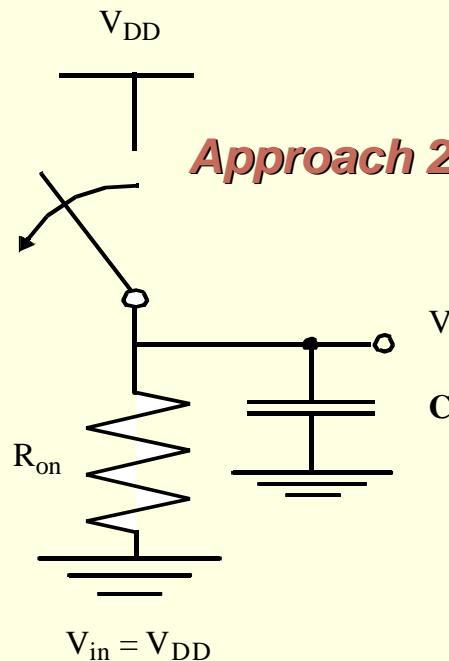
Propagation  
delay - 4



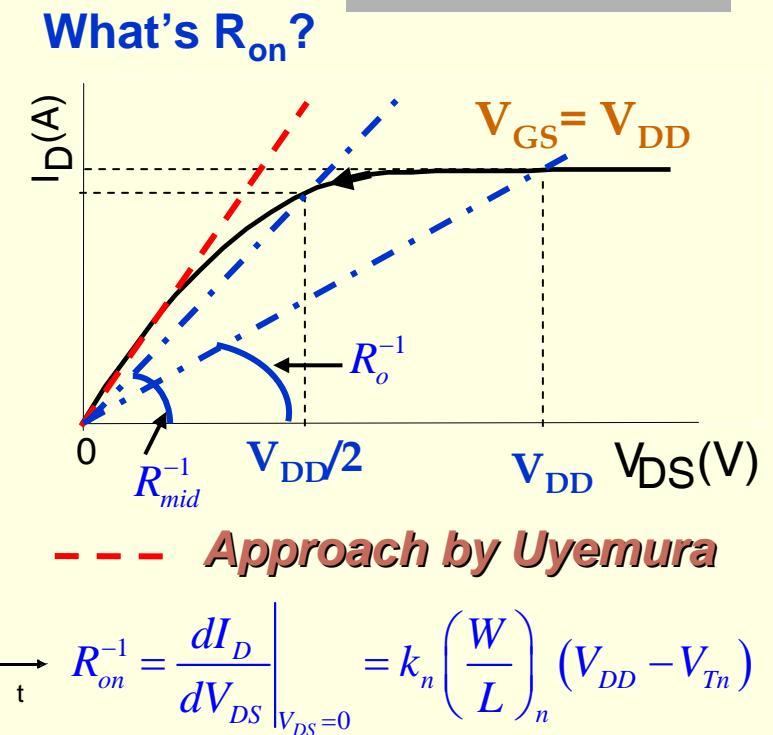
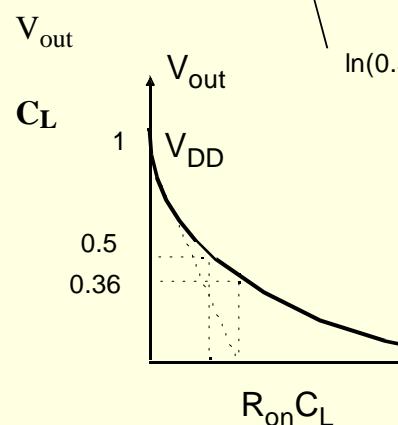
**Figure 5-13** Parasitic capacitances, influencing the transient behavior of the cascaded inverter pair.

# Dynamic operation - 8

## Propagation delay - 5



$$t_{pHL} = f(R_{on} \cdot C_L) \\ = 0.69 R_{on} C_L$$



**Modeling capacitor discharge  
as in an RC circuit!**

Source: Rabaey

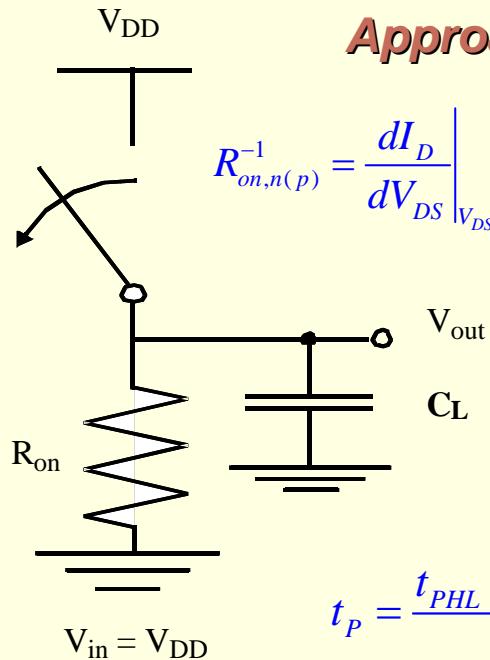
$$t_{pHL} \approx \frac{C}{k_n \left( \frac{W}{L} \right)_n V_{DD}}$$

**Approach by Rabaey**

$$R_{on} \equiv \frac{1}{2} (R_0 + R_{mid})$$

# Dynamic operation - 9

Propagation  
delay - 6



**Approach by Uyemura**

$$R_{on,n(p)}^{-1} = \frac{dI_D}{dV_{DS}} \Big|_{V_{DS}=0} = k_{n(p)} \left( \frac{W}{L} \right)_{n(p)} \left( V_{DD} - (+)V_{Tn(p)} \right)$$

$$\begin{aligned} t_{pHL} &= 0.69 R_{on,n} C_L \\ t_{pLH} &= 0.69 R_{on,p} C_L \end{aligned}$$

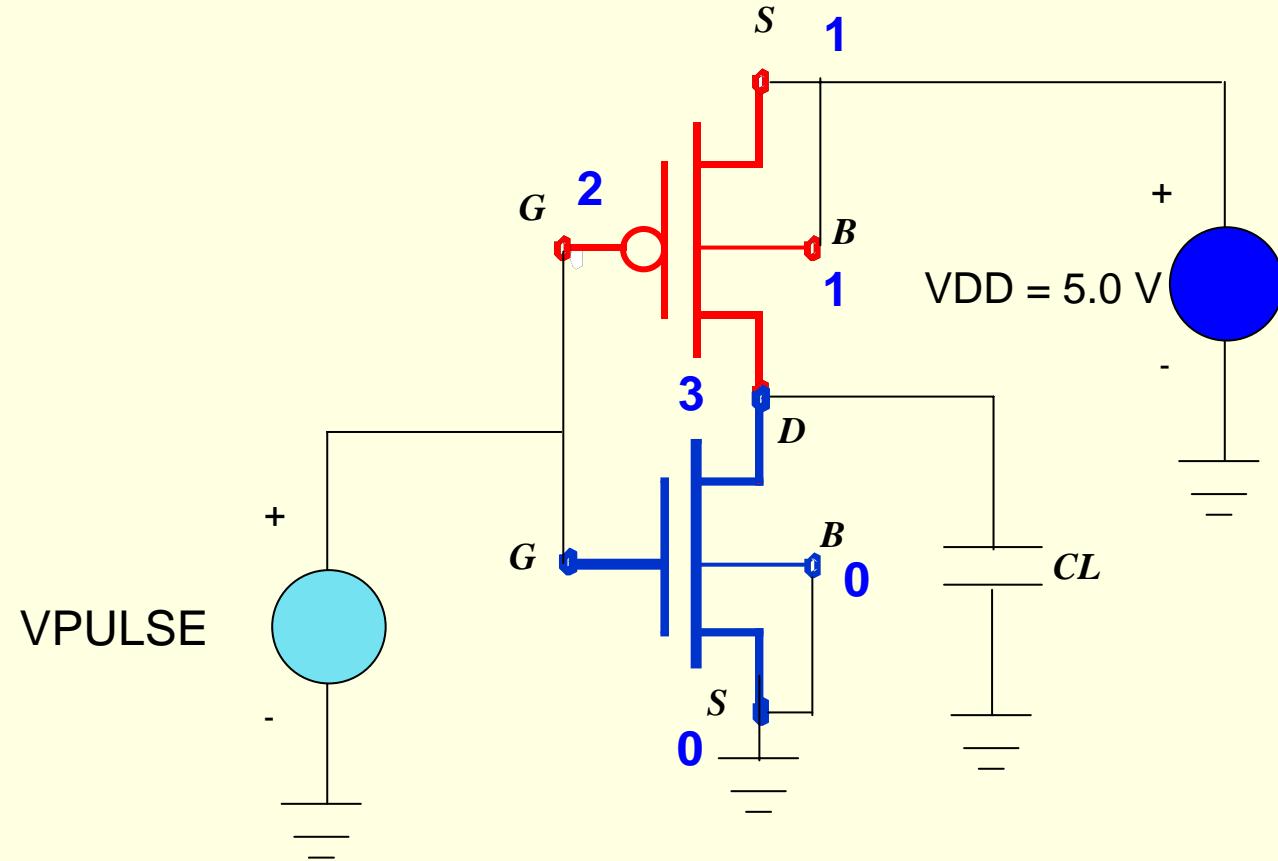
$$t_p = \frac{t_{PHL} + t_{PLH}}{2} = \frac{0.69 \cdot C_L}{2} \left[ \frac{1}{k_n \left( \frac{W}{L} \right)_n \left( V_{DD} - V_{Tn} \right)} + \frac{1}{k_p \left( \frac{W}{L} \right)_p \left( V_{DD} + V_{Tp} \right)} \right]$$

$$t_p \approx \frac{0.69 \cdot C_L}{2V_{DD}} \left[ \frac{1}{k_n \left( \frac{W}{L} \right)_n} + \frac{1}{k_p \left( \frac{W}{L} \right)_p} \right]$$

Source: Uyemura

# Dynamic operation - 10

## Experimental setup



# Dynamic operation - 11

---

## Inverter Propagation Delay

```
* this is the Propagationdelay.cir file
* PMOS transistor description
MP 3 2 1 1 modelp W=2u L=1u
.model modelp pmos (level=1 VT0=-0.65 TOX=7.5n KP=60u lambda=0.0)
* NMOS transistor description
MN 3 2 0 0 modeln W=2u L=1u
.model modeln nmos (level=1 VT0=0.5 TOX=7.5n KP=150u lambda=0.0)
* dc source
vDD 1 0 dc 5.0
*load capacitance
CL 3 0 0.01p
*signal source
v0 2 0 dc 0 pulse 0 5 0 1ps 1ps 200ps 400ps
.end
```

# Dynamic operation - 12

---

**SpiceOpus (c) 6 -> source Propagationdelay1.cir**

**SpiceOpus (c) 7 -> tran 1ps 500ps**

**SpiceOpus (c) 8 -> setplot**

**new     New plot**

**Current tran2     Inverter Propagation Delay (Transient Analysis)**

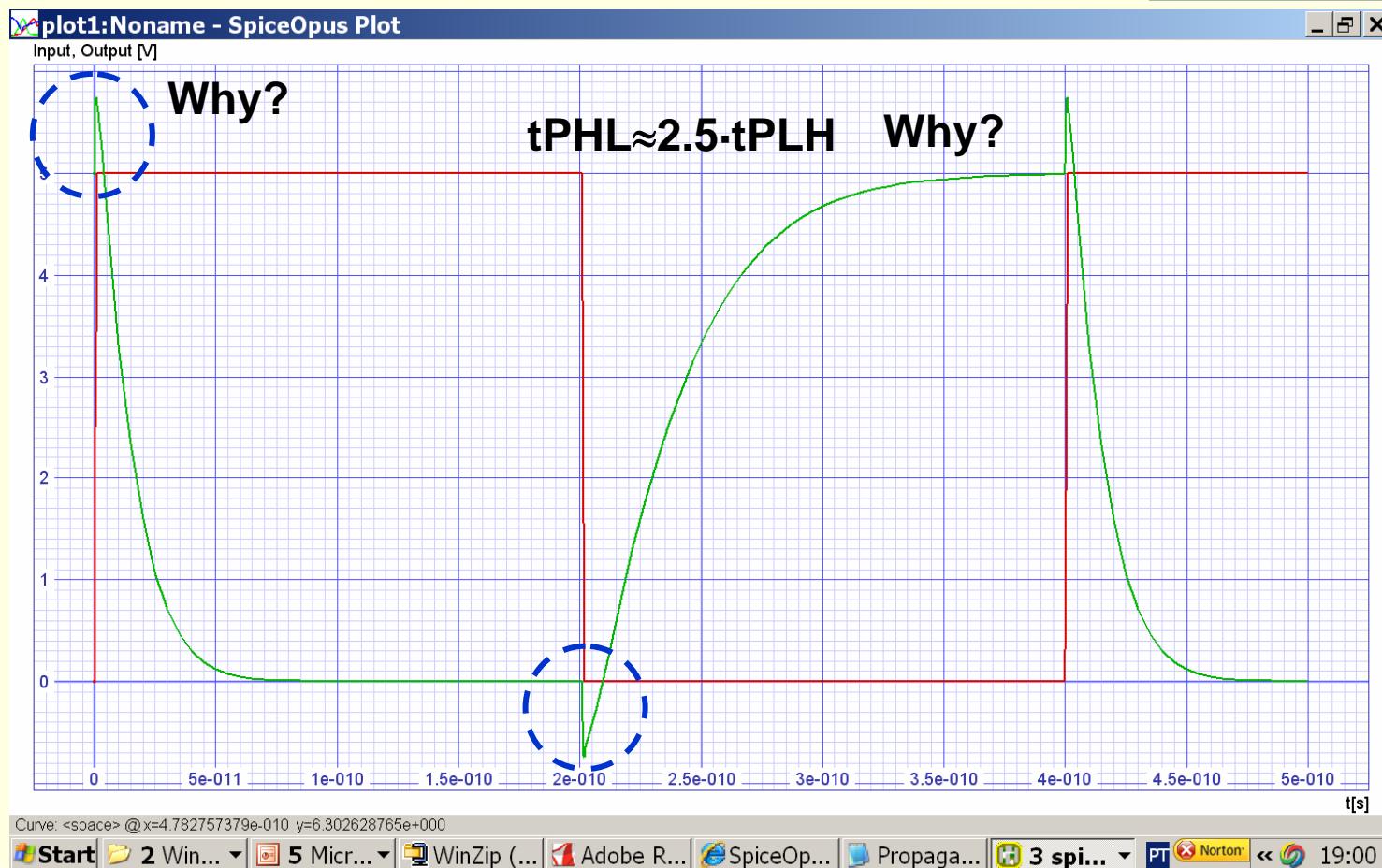
**tran1     Inverter Propagation Delay (Transient Analysis)**

**const     Constant values (constants)**

**SpiceOpus (c) 9 -> setplot tran2**

**SpiceOpus (c) 10 -> plot v(2) v(3) xlabel t[s] ylabel 'Input, Output [V]'**

# Dynamic operation - 12



# Dynamic operation - 14

---

## Exercise

**Simulate the transient response of the inverter of the previous exercise for fan-outs of one and two inverters**

# Dynamic operation - 15

---

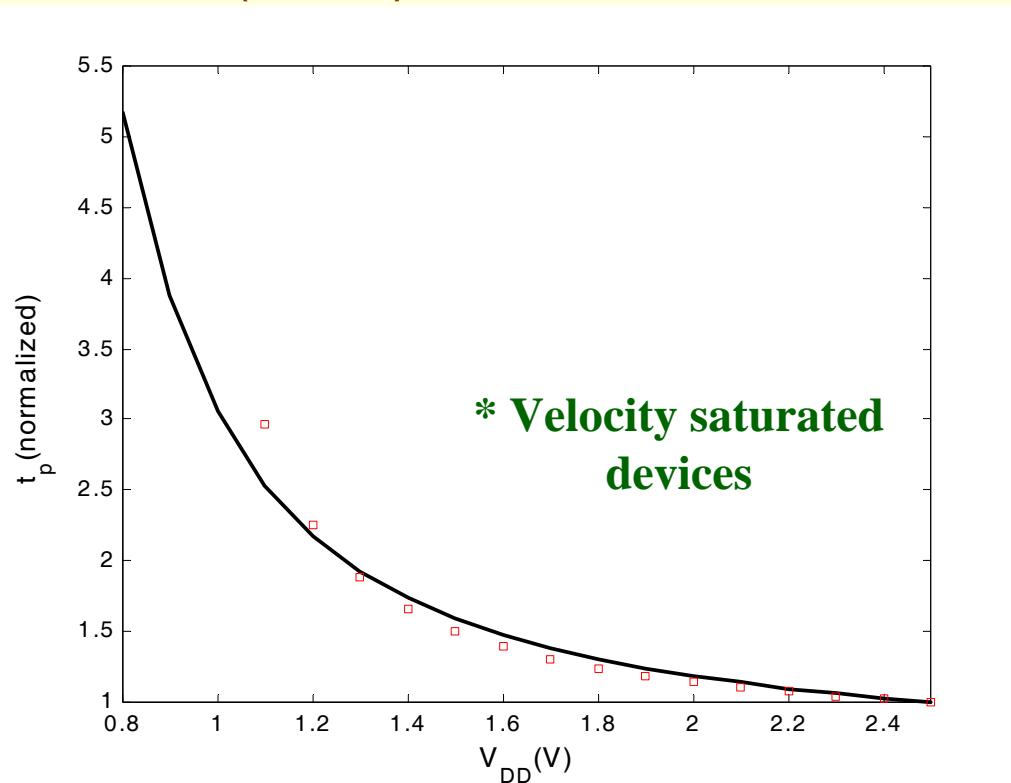
## Design for Performance

- Keep capacitances small
- Increase transistor sizes (W)
  - watch out for self-loading!
- Increase VDD (????)

# Dynamic operation - 16

## Design for Performance

- Increase VDD (????)

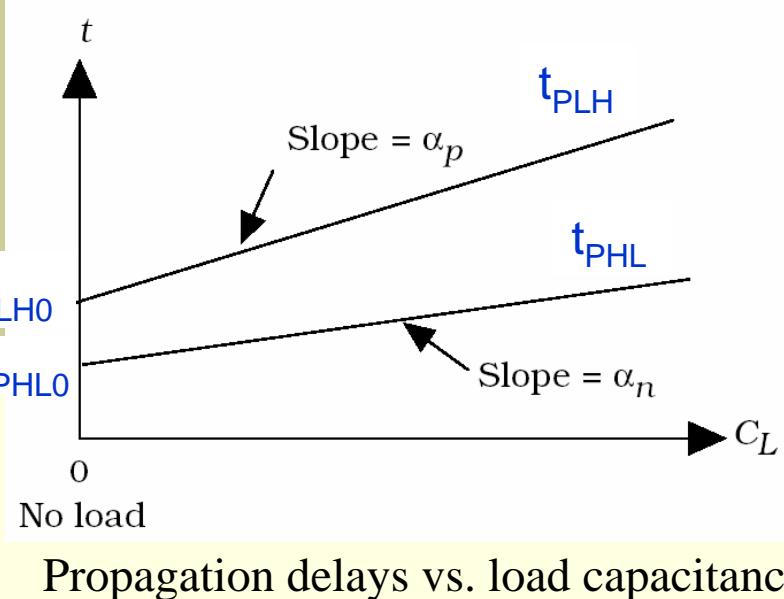


Source: Rabaey

# Dynamic operation - 17

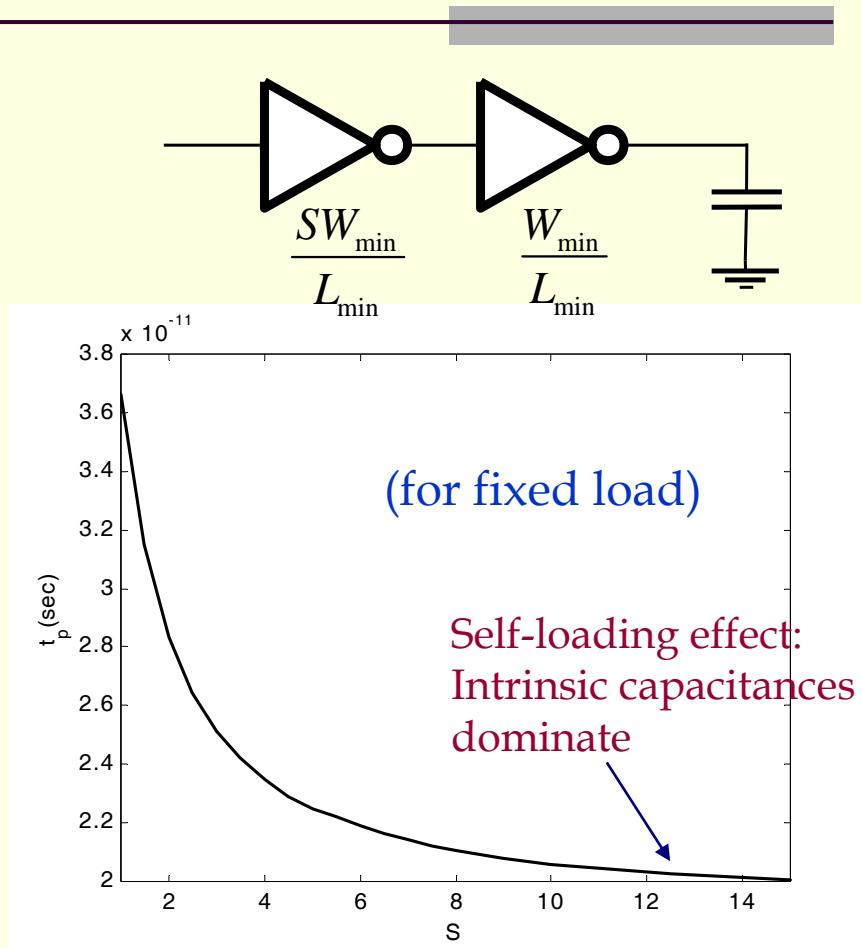
## Design for Performance

- Increase transistor sizes (W)
  - watch out for self-loading



Source: Uyemura

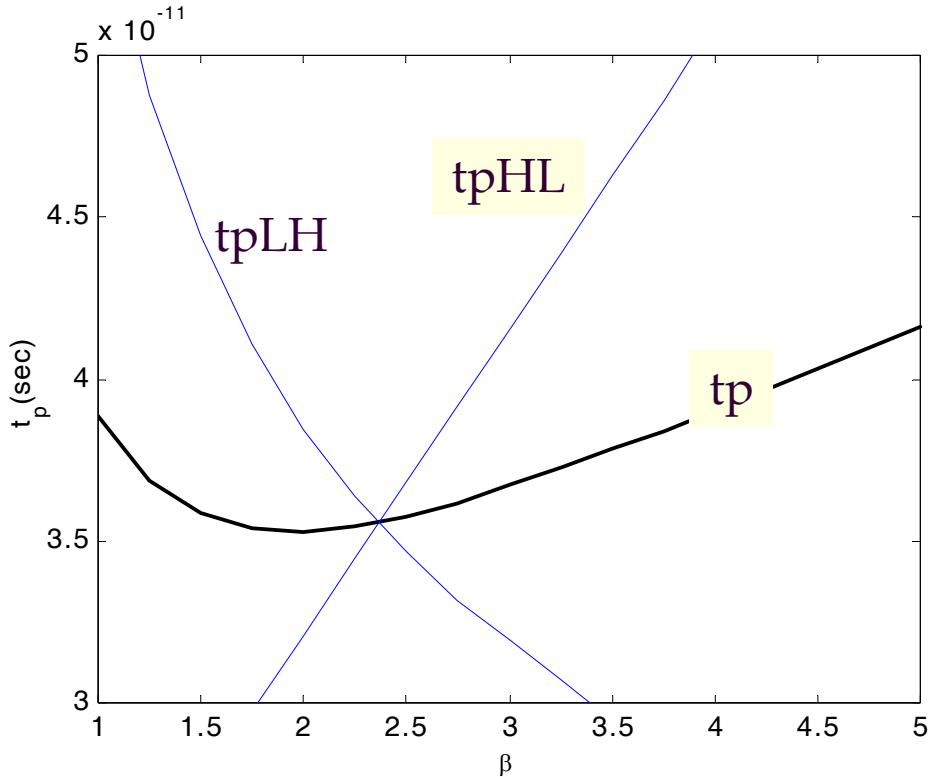
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Digital Integrated Circuits



Source: Rabaey

# Dynamic operation - 18

## Design for Performance

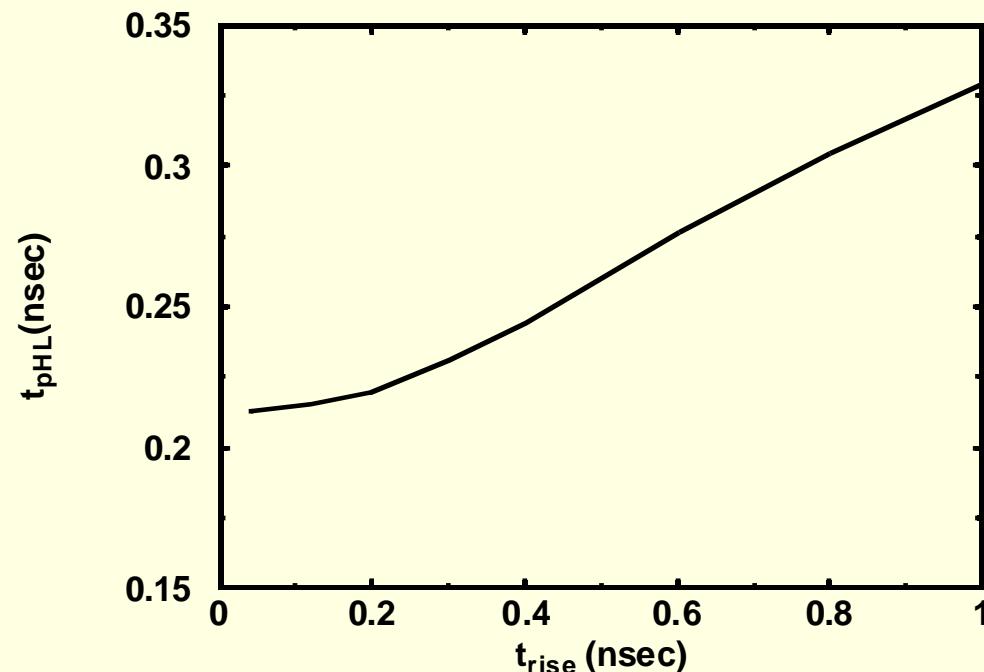


Propagation delays vs. PMOS-to-NMOS  
transistor ratio  $\beta = W_p/W_n$

Source: Rabaey

# Dynamic operation - 19

## *Impact of Rise Time on Delay*



$$t_{pHL} = \sqrt{t_{pHL(step)}^2 + (t_r/2)^2}$$

Source: Rabaey