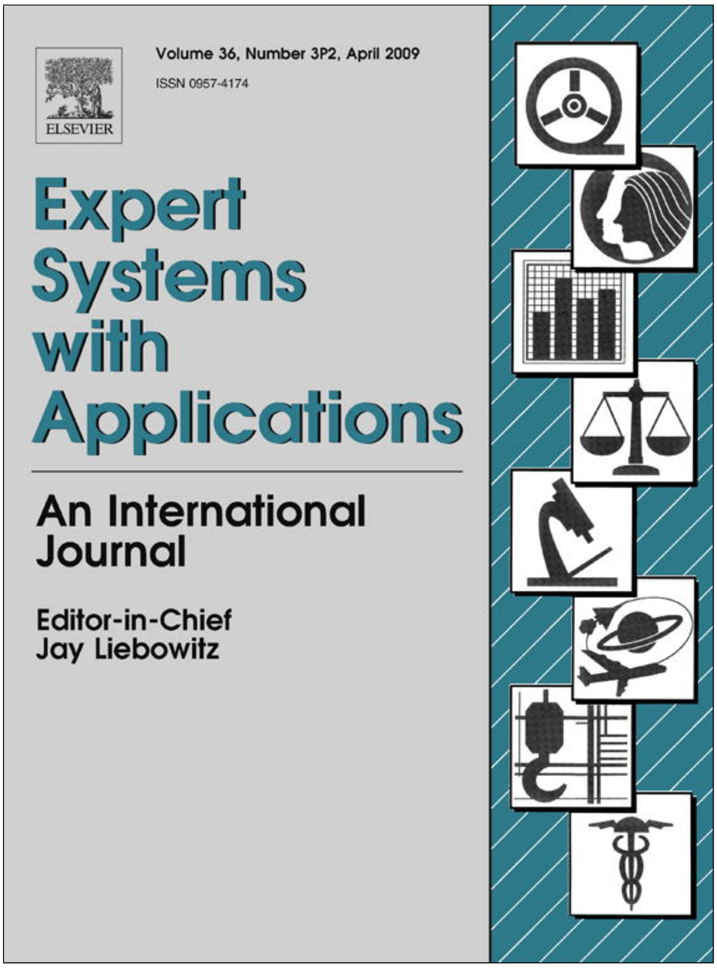
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A hybrid fuzzy-probabilistic system for risk analysis in petroleum exploration prospects

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ABSTRACT

Petroleum exploration is an economical activity where many billions of dollars are invested every year. Despite these enormous investments, it is still considered a classical example of decision-making under uncertainty. In this paper, a new hybrid fuzzy-probabilistic methodology is proposed and the implementation of a software tool for assessing the risk of petroleum prospects is described. The methodology is based in a fuzzy-probabilistic representation of uncertain geological knowledge where the risk can be seen as a stochastic variable whose probability distribution counts on a codified geological argumentation. The risk of each geological factor is calculated as a fuzzy set through a fuzzy system and then associated with a probability interval. Then the risk of the whole prospect is calculated using simulation and fitted to a beta probability distribution. Finally, historical and direct hydrocarbon indicators data are incorporated in the model. The methodology is implemented in a prototype software tool called RCSUEX ("Certainty Representation of the Exploratory Success"). The results show that the method can be applied in systematizing the arguing and measuring the probability of success of a petroleum accumulation discovery.

1. Introduction

Petroleum exploration is an economic activity plenty of decision problems involving risk and uncertainty. As economical and technological resources are limited, managers of petroleum companies frequently face important decisions regarding the best allocation these scarce resources among exploratory ventures that are characterized by substantial financial risk and geological uncertainty. Few years ago, many petroleum companies improved their exploration performance by using principles of risk analysis and portfolio management. According to Rose (2001), in present days, the adoption of standardized risk analysis methods are essential to portfolio management, in order to optimize the allocation of exploration capital.

There are a lot of activities involved in modern petroleum exploration business. Tasks range from modeling geologic theories and data acquisition to econometrics simulations and selection of reservoir, drilling and completion technologies. In this work, we focus our attention to the problem of estimating the chance of success of finding hydrocarbon on a given prospect. That is once an appropriate geological model has been established and an exploration area has been selected, the next step in petroleum exploration is the identification of drilling prospect by geoscientists. This process is critical and requires geotechnical expertise and creativity. After the prospect has been identified, estimating the chance that a producible hydrocarbon accumulation is present, is one of the most important tasks in order to determine the prospect's value.

For many decades of petroleum exploration ventures has been dealt with probability theory as the formal tool to handle and represent uncertainty quantities (da Silva, 2000). Such representation are usually expressed as a probability value, known as "probability of success" (geological or economical), found by the combination of other probabilities that represent, the assessment of geological factors as source rock, trap, reservoir and seal, considered individually and combined by traditional and numerical methods as Monte Carlo Simulation (Behrenbruch, Turner, & Backhouse, 1985; Newendorp & Schuyler, 2000; Rose, 1992).

Despite the great progress in economical risk analysis and portfolio management, the "probability of geological success", i.e., the discovery of a hydrocarbon accumulation in a given exploratory prospect, is still a new and very hard area of research.

Uncertainty is intrinsically involved in all petroleum venture predictions, and particularly in chance of discovery (Rose, 2001). The problem is how to express the technical uncertainties realistically, and in a form that can be used in economic equations in order to estimate the economical risk (Rose, 2001). Geologist suffer in trying to reduce very complex and uncertain knowledge in just a single few numbers that represent the exploratory chance of success.

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There are extensive attempts in systematizing the process of correctly estimate chance of success of finding hydrocarbon on a given prospect (MacKay, 1996; Newendorp & Schuyler, 2000; Otis & Schneidermann, 1997; Rose, 2001). But, as this process is done by different geoscientists, in different geologic areas and under a very competitive scenario, it frequently leads to optimistic or pessimistic bias in the prospectors estimative (Rose, 2001).

The bias is a very important problem in prospect risk assessment. If the prospect chance of discovery or economic value are contaminated with biased estimates, the exploration company's decision investments will lead to suboptimal economic performance (Rose, 2001). The more relevant type of bias that affect judgment under uncertainty are Overconfidence – predictive ranges are too narrow, indicating that estimators are much less accurate than they think they are; Overoptimism – prospectors exaggerate magnitude of reserves or chance of success in order to sell the deal; and Representativeness – analog based on small sample size may not be statistically significant (Rose, 2001).

The fuzzy set theory has been used to represent and solve problems of petroleum evaluation. Chen and Fang (1993), Chen, Osadetz, Embry, and Hannigan (2002) and Tounsi (2005) use fuzzy logic and approximate reasoning to asses petroleum field in different regions. In most of these studies, the geological factors are coupled with multiple-criteria decision-making theory. However, this approach has some inconveniences: the incorporation of *a posteriori* knowledge as historical and direct hydrocarbon indicators data cannot be easily incorporated in the system, and the difficulty to incorporate qualitative expressions like "excellent", "fair" or "poor" in the economical evaluation formulas.

In this paper, we present a new fuzzy-probabilistic methodology capable to represent uncertain geological knowledge and the prototype software tool called RCSUEX ("Certainty Representation of the Exploratory Success") that implements the methodology (Schoeninger, 2003). The main purpose of this work is to provide a method to deal with the problem of systematizing the process of correctly estimate chance of success of find hydrocarbon on a given prospect and to facilitate and to standardize the geologist argumentation task. This fuzzy-probabilistic methodology is founded in the following assumptions: risk can be qualified by set of questions and answers concerning the decision problem (Hardman & Ayton, 1997); when expressions like "moderate" and "severe" are significant for the domain expert, then fuzzy sets are more suitable for knowledge representation than "classical" or crisp sets (Terano, Asai, & Sugeno, 1994); fuzzy logic is adequate to represent uncertainty in petroleum geology (Chen & Fang, 1993; Fang & Chen, 1990); the beta probability distribution is pertinent to represent the certainty of success of a random variable in a Bayesian approach (Groot, 1970).

The paper is organized as follows: in Section 2, describes how risk analysis can be applied in the petroleum exploration process focusing in the elements of the hydrocarbon system and estimating the chance that a subsurface trap exist and if it is capable to store and accumulate hydrocarbons. Section 3 presents how fuzzy reasoning can be used as a very efficient mechanism to deal with incomplete and imprecise data, and knowledge expressed in vague and linguistic terms that characterize the petroleum risk evaluation problem. In this section, fuzzification, rule evaluation and defuzzification are described separately and particularities specific for the problem are discussed. Section 4 describes the process performed in our system (RCSUEX) to map from a subjective fuzzy domain to a objective probabilistic domain. Section 5 explains the importance and the methodology to incorporate historical data and Direct Hydrocarbon indicators in order to improve risk assessment. This section proposes a mathematical model that put together the objective perspective with the subjective one. Section 6 shows the application of the proposed method for a simple prospect risk assessment. Section 7 gives conclusions and brief discussion on the proposed methodology.

2. Risk analysis in the petroleum exploration process

The petroleum exploration process is highly coupled with geological models that explain the occurrence of hydrocarbon accumulations. Geologists, geophysicists and seismologists apply high levels of expertise to answers questions such as: what is the chance of finding an accumulation in the prospect? What is the volume of the accumulation? Which method should be used to recover the petroleum from the field? Capturing this knowledge and representing it in a formal model is a permanent aim for knowledge management in petroleum companies (Tounsi, 2005).

For many decades petroleum companies have assessed risky projects involving uncertainties about positive monetary results – predicting the distribution of financial gains or losses that may result from the drilling of an exploration well through objective procedures and principles of statistics, probability and utility theories (Harbaugh, Davis, & Wendebourg, 1995). In this study, we focus our attention, evaluating geological factors (subjective data) incorporated with statistical information (objective data). The information used in this evaluation came from usual seismic data, analogies and geological theories. Probability-analysis methods have been developed which make use of widely available forms of exploration information. Geophysical data, subsurface information derived from well logs, and production data can be analyzed by statistical methods to yield objective forecasts expressed as probabilities (Harbaugh, Doveton, & Davis, 1977).

According to Otis and Schneidermann (1997) in 1989, Chevron Overseas Petroleum Inc. developed a process to allow management to compare a wide variety of global exploration opportunities on a uniform and consistent basis. The final product was a continuous process that integrates geologic risk assessment, probabilistic distribution of prospect hydrocarbon volumes, engineering development planning, and prospect economics. The process was based on the concepts of the play and hydrocarbon system. Our work is also based in the play and hydrocarbon system concepts, but we focus mainly in obtaining the probability of geologic success i.e., if a stabilized flow of hydrocarbons is obtained on test of a exploratory well.

The hydrocarbon system concept can be used as an investigating model for hydrocarbon discoveries as it describes the geologic relationship between elements and processes since the play source rocks, reservoir, and seal until the result as oil or gas accumulations. Essential geologic elements of the hydrocarbon system are

- play source rock;
- reservoir rock and
- seal rock.

While the geologic processes of the hydrocarbon system are

- trap formation;
- genesis-migration and
- timing-synchronicity.

Information and data from each of these factors are collected and analyzed by geoscientist and engineers that then consider the "*favorability*" or the probability of success of each of these six elements of the play concept. Multiplication of these probabilities yields the probability of geologic success of the prospect (Otis & Schneidermann, 1997). Obviously, this mapping from the qualitative thought into a probabilistic number is a very hard task to geoscientists since all argumentation is subjective.

According to Alexander and Lohr (1998), even considering known facts and past experience, explorationists tend to be conservative or optimistic when estimating chance of success. Projects that have a moderate estimative (25–65%) chance of success are often successful about 35–75% of the time. While "high-risk" projects that have a less than 20% estimated chance of success have found oil in less then 5% of the time.

Taking into account only geologic reasoning and neglect the observed frequency of success, means to ignore historic data from the area concerning the geologic concept being studied. The same way, it is a great mistake to observe only historical data and ignore collected information as seismic, geologic argumentation and hydrocarbon indicators.

The incorporation of *a posteriori* knowledge as historical and direct hydrocarbon indicators data into the probability of geologic success of the prospect brings the explorationists estimative closer to the real chance of success.

3. Fuzzy expert systems

Risk assessment in petroleum exploration prospects is a process that takes into account different geological and statistical information, imprecise and incomplete by nature. Analyzing the expert reasoning mechanism, we observe that the decision involves aggregation of imprecision and incomplete variables through the use of linguistic terms in a process called approximate reasoning (Dubois & Prade, 1984).

Traditional symbolic expert systems fail to model this approximate reasoning and to deal with incomplete and imprecise data.

The fuzzy set theory and the fuzzy logic introduced by Gaines, Zadeh, and Zimmerman (1984), Zadeh (1965), Zadeh (1971) and Zadeh et al. (1975) seems to be the appropriate approach to model human experts reasoning process much better than conventional expert systems (Nafarieh & Keller, 1991). Among many advantages of fuzzy set theory we can evidence: the ability to deal with ill defined class boundaries; the ability to model approximate reasoning decisions based in fuzzy linguistic variables (low, good, and high) using fuzzy set operators (and, or); and the possibility of making useable non-numeric (e.g. qualitative) information (Tounsi, 2005). All these evidences make the fuzzy approach a very efficient mechanism to deal with an incomplete and imprecise data, and knowledge expressed in vague and linguistic terms that characterize the petroleum evaluation problem.

In our system, we developed a Fuzzy Expert System to evaluate the "*favorability*" or the probability of success of each of the six elements of the play concept. Each element (*play source rock, reservoir rock, seal rock, trap formation, genesis-migration and timing-synchronicity*) is evaluated individually by a series of fuzzy rules extracted from a group of domain experts in a process described in the following subsections.

The Fuzzy Expert System uses fuzzy reasoning (also known as approximate reasoning) as the inference process of formulating the mapping from a given input to an output using a set of fuzzy if–then rules (Jang & Sun, 1995). The mapping then provides a basis from which decisions can be made. The process of fuzzy inference involves in general three operations:

- Fuzzification: Translation from real world values to fuzzy values.
- Rule evaluation: Computing rule strengths based on rules and inputs.
- Defuzzification: Translate results back to the real world values.

Mamdani's fuzzy inference method is the most commonly seen fuzzy methodology. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed in 1975 by Ebrahim Mamdani (Mamdani, 1976). Mamdani-type inference expects the output membership functions to be fuzzy sets. After the aggregation process of rule evaluation operation, there is a fuzzy set for each output variable that needs defuzzification.

3.1. The fuzzification

Fuzzy set theory permits the gradual assessment of the membership of elements in a set and so offers a formalism to describe such linguistic variables in the form of fuzzy sets. The concept of grades of membership or the concept of possibility values of membership is used (Zadeh, Fu, Tanaka, & Shimura, 1975; Zadeh, 1989) to represent the shades of meaning of such linguistic terms. The membership defines how each point in the input space is mapped to a given set. Membership value (or degree of membership) is usually a real number between 0 (non membership) and 1 (full membership). We write $\mu(x)$ to represent the membership of some object x from the universe of discourse X in the fuzzy set A, thus:

$$\mu_A(x) : X \rightarrow [0, 1] \tag{1}$$

Any fuzzy set may be described by a continuous mathematical function or discretely by a set of pairs of values (numeric values of linguistic variable, and corresponding grades of membership) (Tounsi, 2005) as

$$F_A = \{(x, \mu_A(x)) \mid x \in X\} \tag{2}$$

where $\mu_A(x)$ defines a grade of membership of variable x in the fuzzy set A.

The first operation in the Fuzzy Expert System is fuzzification, which converts each piece of input data to degrees of membership by a lookup in one or several membership functions. The fuzzification operation thus matches the input data with the conditions of the rules to determine how well the condition of each rule matches that particular input. There is a degree of membership for each linguistic term that applies to that input variable (Bogenberger & Keller, 2001). In short, fuzzification makes the translation from real world values to fuzzy world values using membership functions.

In petroleum evaluation it is often not easy to fix the input elements that determine the condition of the output variable, and in many cases, the boundary between different classes of values of an input element is very hard or even impossible to determine. In such cases, fuzzy set theory can be applied to deal with the uncertainty of the classification given by the explorationist. For example, when asked to classify a qualitative variable, like"kind of trap", the geologist enters with the class of trap he thinks exists in trap formation and with the certainty or confidence he has in that response.

In order to deal with these non quantifiable input variables that determine geological elements and processes of the hydrocarbon system during the fuzzification operation, we classify the input variables as categorical (nominal), ordinal and interval (or quantitative). For each kind of variable we apply a slightly different fuzzification operation.

3.1.1. Fuzzification of categorical variables

A categorical variable (sometimes called a nominal variable) is one that has two or more categories, but there is no intrinsic ordering to the categories. For example, gender is a categorical variable having two categories (male and female) and there is no intrinsic ordering to the categories.

Some input variables in the hydrocarbon system concept modeling are categorical in nature. For example, the kind of closure used to determine the trap element can be classified as stratigraphic, structural or combination trap. The different categories of this categorical variable does not keep any ordered relation between each other and could be considered as independent singular variables (or singletons) and not fuzzy sets. But in many cases the geologist does not have absolute certainty of its categorization. So, in RCSUEX, when the geologist answer the class of a categorical variable, he also supplies his level of confidence in the response as a number between 0% and 100%. This confidence level becomes the input for fuzzification of the variable. In cases of 100% confidence in response, the degree of membership to the class is 100% (full membership). For other confidence levels, the degree of membership to the class is given by the confidence level itself, but the remainder is equally distributed among the other classes of the variable. The justification for this decision is to allow the evaluation of inference rules involving the other classes of the variable (Schoeninger, 2003). In Fig. 1 shows the fuzzification of kind of closure where the geologist assess that a structural closure is involving the reservoir objective and of minimal adequate area with a confidence level of 50% resulting in a 0.5 membership value. Note that remainder of confidence is equally distributed between the other two kind of closures.

3.1.2. Fuzzification of ordinal variables

An ordinal variable is similar to a categorical variable. The difference between the two is that there is a clear ordering of the variables even in the absence in many cases of a unit of measure to that variable. Moreover, the spacing between the values may not be the same across the levels of the variables. For example, the path visibility when assessing the migration factor is an ordinal variable that can be categorized as poor, not-so-good, good and very good. Despite we order the categories, there is not a unit of measure to precisely quantify quality of the path visibility in the prospect migration factor.

In the input interface of RCSUEX, if the variable is classified as ordinal, the geologist uses a sliding-bar component, representing real numbers between 0 and 100, to enter the "value" of the ordinal variable. After that, the fuzzification is done based in this value as shown in Fig. 2.

3.1.3. Fuzzification of rational variables

A rational or quantitative or numerical variable is similar to an ordinal variable, except that we can associate an unit of measure to the variable and the intervals between the values are equally spaced. For example, the depth where we expect to find the source rock can be categorized as shallow, medium or deep, but there are a unit of measure (meters or feet) and a numerical value to associate with the variable. In this case, this numerical value is used to calculate the membership value to the linguistic categories in the usual way as shown in Fig. 3.

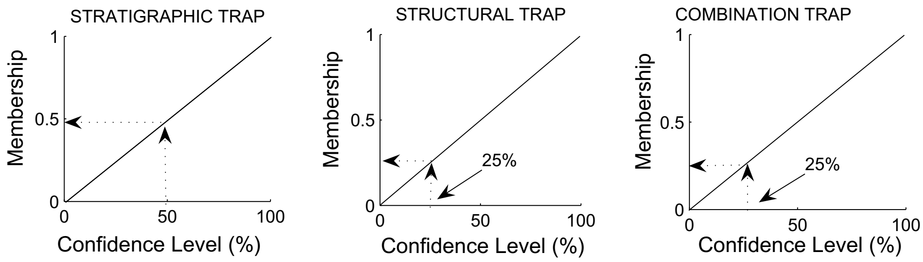

Fig. 1. Fuzzification of a categorical variable with confidence level.

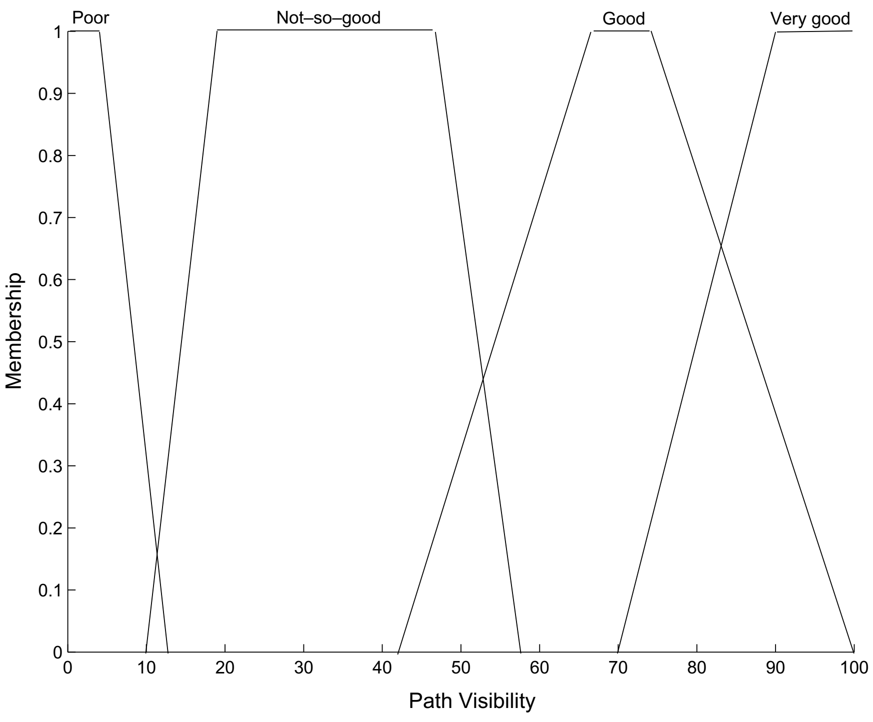

Fig. 2. Ordinal variable membership function.

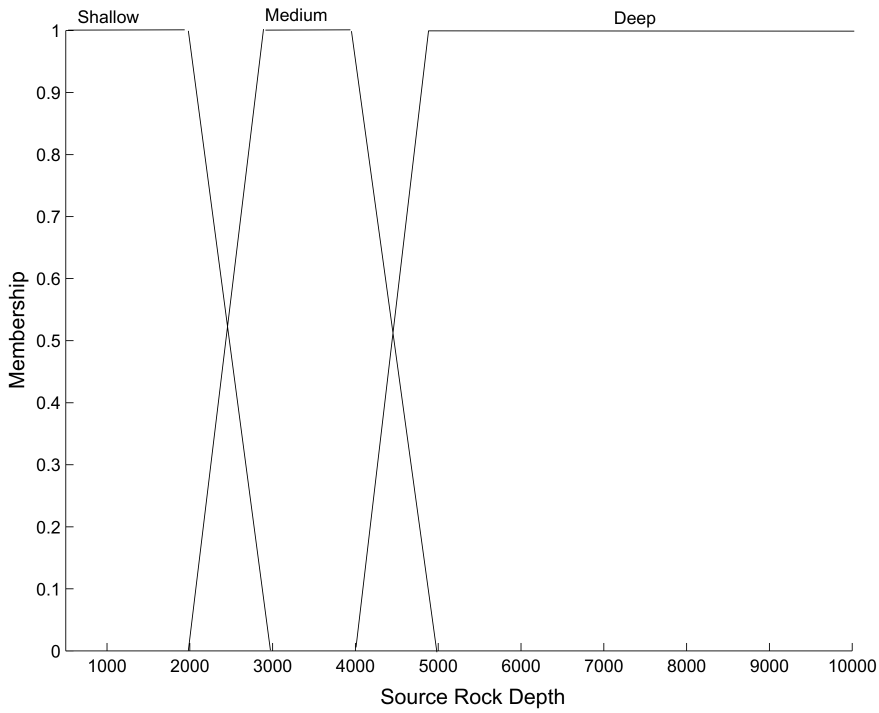

Fig. 3. Rational variable membership function.

3.2. Rule evaluation

The rules, also known as the knowledge base, are the core element of the fuzzy expert system. Rules can be obtained based on expert opinions and system knowledge. Generally, a fuzzy rule follows an IF <premise> THEN <consequent> format, including the possibility to combine several premises with logical operators in the form IF <premise 1> AND/OR <premise 2> AND/OR <premise 3>...THEN <consequent>.

Rule evaluation, based on fuzzy set theory, uses fuzzy operators to perform logical operations such as the complement, intersection, and union of sets.

With the fuzzification of inputs, we can know the degree to which the antecedent has been satisfied for each rule. In cases where the antecedent of a given rule has more than one premise, the fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule. The fuzzy operator perform logical operations such as the complement, intersection, and union of sets. In the RCSUEX system they are defined as follows:

- Complement or logical NOT that corresponds to the degree of truth of the membership to the complement of the set is defined as

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{3}$$

- Intersection is analogous to the logical AND operator and is defined as the smaller of the memberships in the sets

$$\mu_{\text{combination}} = \text{MIN}(\mu_A, \mu_B, \mu_C, \ldots) \tag{4}$$

where μ_A, μ_B, μ_C are the fuzzy membership value for premises *A*, *B*, *C* and so on.

- Union or logical OR operator is defined as the larger of the memberships in the sets.

$$\mu_{\text{combination}} = \text{MAX}(\mu_A, \mu_B, \mu_C, \ldots) \tag{5}$$

where μ_A, μ_B, μ_C are the fuzzy membership value for premises *A*, *B*, *C* and so on.

The result of the logical operators is a number to be applied to the output variable. If two different rules produce similar outputs a further reduction method is necessary. The maximum method of rule deduction takes the maximum degree of membership for the output, since this corresponds to the union of two output sets.

3.3. Defuzzification

In most applications, crisp results are required instead of fuzzy ones. So, usually the last operation in a Fuzzy Expert System is the defuzzification, used to transform the fuzzy inference results into a crisp output.

According to Jiang and Li (1996), defuzzification can be mathematically interpreted as a mapping strategy, implemented by a decision-making algorithm, from a fuzzy set *F* on an universe of discourse *X* into a designated crisp value from the space *X*. It is expected that the crisp value is significant with respect to *A*, i.e., that it best represents the fuzzy set *A* as

$$F^{-1}(A) : \{\mu_A(x) \mid A \in F(X), x \in X\} \to X. \tag{6}$$

Runkler (1997) assumed that the significant element can be determined from a fuzzy result by a human user, although multiple possibilities may exist. Applications usually work with the centroid algorithm whose calculation is

$$F^{-1}_{\text{COG}}(A) = \frac{\int_X \mu_A(x) \cdot x \, dx}{\int_X \mu_A(x) \, dx} \tag{7}$$

As in our application each element of the petroleum system is assessed in order to supply the success rate of occurrence of the element, it seems appealing to interpret such probability with fuzzy theory. Despite some confusion can be present in the way the definitions are understood, it is very important to distinguish fuzzy models from statistical models since they represent rather different kinds of information (Dubois & Prade, 1993).

In the same way, Dubois and Prade (1993) stated that some interpretations of fuzzy sets are in agreement with probability calculus. There are many studies and different proposals of such interpretations: Cheng and Agterberg (1999), in their paper defined 'fuzzy probability' in terms of fuzzy membership values and used it to derive 'fuzzy posterior probability' of mineral deposits. The model, as much probabilistic as fuzzy, uses a data-driven approach for calculating fuzzy membership values.

Gettings and Bultman (1993) applied the fuzzy set theory for quantification of favorableness for a mineral resource appraisal. The possibility of each condition necessary for the formation of the deposit was represented as fuzzy sets. The intersection of the fuzzy sets measure the degree of simultaneous occurrences of the necessary factors and provides a fuzzy favorability map of the deposit occurrence. Carranza and Hale (2001) used a similar technique to generate fuzzy predictive maps of gold mineralization potential combining fuzzy sets of favorable distances to geological features and favorable lithologic formations with a fuzzy logic inference engine.

Porwal (2006) also used a data-driven approach in which the theory of probability was used to calculate weights of evidence for deriving fuzzy membership values. However, the fuzzy values calculated do not represent probability of mineral occurrence but only indicate favorability in a relative sense. The described model then combines multi-class predictor maps and generate mineral potential maps based on a Bayesian probability framework.

Chen and Fang (1993) attribute the following linguistic values to an exploratory prospect as a whole, in terms of its favorability: "*excellent*", "*very good*", "*good*", "*fair*" and "*poor*". In RCSUEX implementation, there are a fuzzy inference system to evaluate the condition of occurrence of each geological factor of the exploratory prospect, regarding its possibility of occurrence or favorability. The output variable of each inference system is interpreted as favorability and is composed by the following linguistic expressions: "***favorable***", "***encouraging***", "***neutral***", "***questionable***" and "***unfavorable***". Like probability, the variable range is between [0 and 1]. The defuzzification process uses the centroid method to convert the possibility distribution associated with the defuzzified output value with a probability distribution satisfying some conditions and can be interpreted as a '*fuzzy a priori probability*' or '*fuzzy subjective probability*'. Fig. 4 shows the favorability fuzzy sets defined for each geological factor.

As the favorability map is generated using the centroid of area method by a knowledge-driven inference that depends on the quality of the available data and the proper geologic modeling for each factor, the favorability representation of each factor as a probability distribution is much more appropriate to represent the imprecisions and uncertainties of the process and to connect to the data-driven step where historical data are incorporated into the model. The process to transform the favorability map into a probability distribution is explained in the next section.

4. Mapping from subjective favorability to probabilistic distribution

In this section, we describe the process performed in our system to map from a subjective fuzzy domain to a objective probabilistic domain. This process in compound by three steps: the generation of a uniform probability distribution associated with the favorability of each geological factor; the combination of these probability distributions through a Monte Carlo simulation to generate the prospect probability as a whole; and finally the adjustment of the simulated samples to a Beta probability distribution.

4.1. First step: generating a uniform probability distribution

The first step in this process corresponds to map the favorability value of each geological factor obtained by the fuzzy inference engine to an uniform probability distribution. The idea here is calculate the probability distribution associated with the *fuzzy subjective probability* interpretation. The uniform distribution was adopted as an *ad hoc* alternative in order to simplify the computation of the

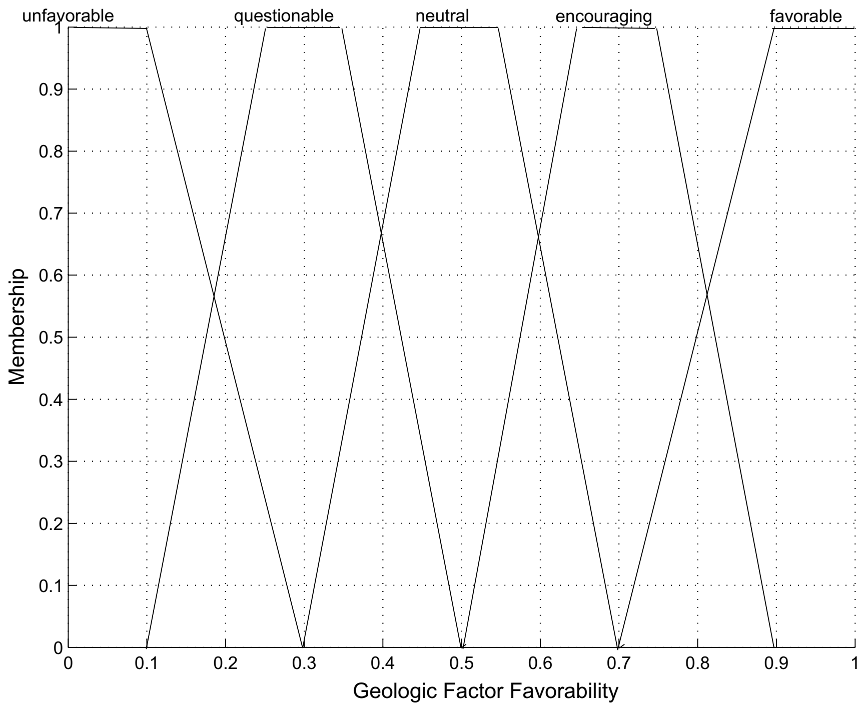

Fig. 4. Favorability membership function for each geological factor.

interval, but other probability distributions could be examined. In our system, as we used symmetric trapezoidal membership function for each favorability fuzzy set described as

$$F(x) = \begin{cases} L\frac{(a_L - x)}{\alpha} & \text{if } x \in [a_L - \alpha, a_L] \\ 1 & \text{if } x \in [a_L, a_R] \\ R\frac{(x - a_R)}{\alpha} & \text{if } x \in [a_R, a_R + \alpha] \\ 0 & \text{if otherwise} \end{cases} \tag{8}$$

where $[a_L, a_R]$ is the core of F, and $(a_L - \alpha), (a_R + \alpha)$ is the support of F.

By this way, we map this favorability value to a uniform distribution into the $[a_L - \alpha, a_R + \alpha]$ interval, were the defuzzified output value corresponds to the mean value of the interval. When the favorability value belongs to two favorability fuzzy sets, an interpolation is necessary in order to generate the probability distribution interval as shown by Eq. (9) and Fig. 5

$$\eta = \frac{x - \left(\frac{a_{1R} + a_{1L}}{2}\right)}{\left(\frac{a_{2R} + a_{2L}}{2}\right) - \left(\frac{a_{1R} + a_{1L}}{2}\right)} \tag{9}$$

where x is the favorability value, $[a_{iL}, a_{iR}]$ is the core of each favorability fuzzy set and η is the proportionality constant used to calculate the interpolated uniform probability distribution interval $[l, u]$ as follows:

$$\begin{aligned} l &= (a_{1L} - \alpha_1) + \eta[(a_{2L} - \alpha_2) - (a_{1L} - \alpha_1)] \\ u &= (a_{1R} + \alpha_1) + \eta[(a_{2R} + \alpha_2) - (a_{1R} + \alpha_1)] \end{aligned} \tag{10}$$

In the following example, this calculation is shown for an hypothetical case illustrated in Fig. 5. Suppose that a favorability value of $x = 0.55$ was obtained from the defuzzification step of the fuzzy inference system and that the "*neutral*" and "*encouraging*" trapezoidal fuzzy sets are defined as follows:

$$\text{neutral} = \begin{cases} a_{1L} = 0.45 \\ a_{1R} = 0.55 \\ \alpha_1 = 0.15 \end{cases}$$

$$\text{encouraging} = \begin{cases} a_{2L} = 0.65 \\ a_{2R} = 0.75 \\ \alpha_2 = 0.15 \end{cases}$$

then η is

$$\eta = \frac{0.55 - \left(\frac{0.55+0.45}{2}\right)}{\left(\frac{0.75+0.65}{2}\right) - \left(\frac{0.55+0.45}{2}\right)} = 0.25$$

Now we can calculate the interpolated uniform probability distribution interval $[l, u]$ as

$$\begin{aligned} l &= (0.45 - 0.15) + 0.25[(0.65 - 0.15) - (0.45 - 0.15)] = 0.35 \\ u &= (0.55 + 0.15) + 0.25[(0.75 + 0.15) - (0.55 + 0.15)] = 0.75 \end{aligned}$$

4.2. Second step: sampling generation and Monte Carlo simulation

Bearing in mind that a prospect will be a success if and only if all the geologic factors will be successful, we combine the geologic chance factors of the prospect as usually made by the petroleum industry (Rose, 1992), using the Monte Carlo simulation technique in order to get the joint probability of success. Doing so, we get a non-parametric distribution of all possible chance of success measures in the evaluated prospect.

In the implementation of this step in RCSUEX we generate 1000 pseudo-random numbers with uniform probability distribution in the $[l, u]$ interval determined by the previous step for each geologic factor. Running a Monte Carlo simulation with these samples of success for each geological factor and plotting the result as a histogram we can observe the success probability distribution for the prospect as a whole as shown in Fig. 6.

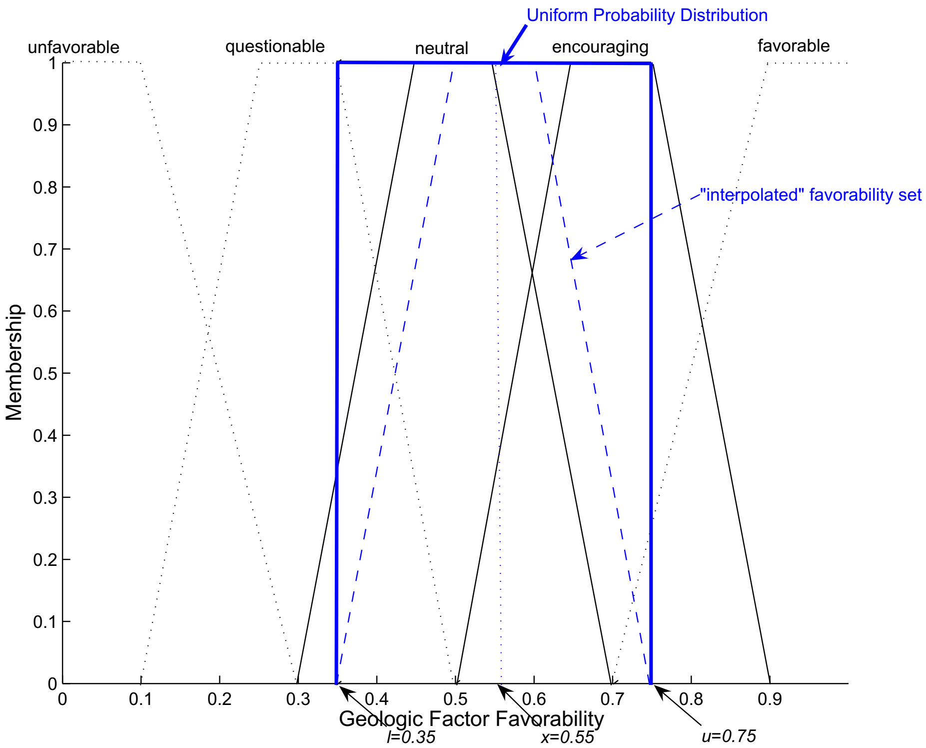

Fig. 5. Generating a uniform probability distribution.

Geologic Factor	Linguistic Favorability	x	l	u
Synchronicity	encouraging/favorable	0.758	0.546	0.923
Trap	neutral/encouraging	0.638	0.438	0.838
Migration	encouraging	0.7	0.5	0.9
Source Rock	neutral/encouraging	0.573	0.373	0.773
Reservoir Rock	neutral/encouraging	0.575	0.375	0.775
Seal Rock	encouraging/favorable	0.66	0.46	0.86

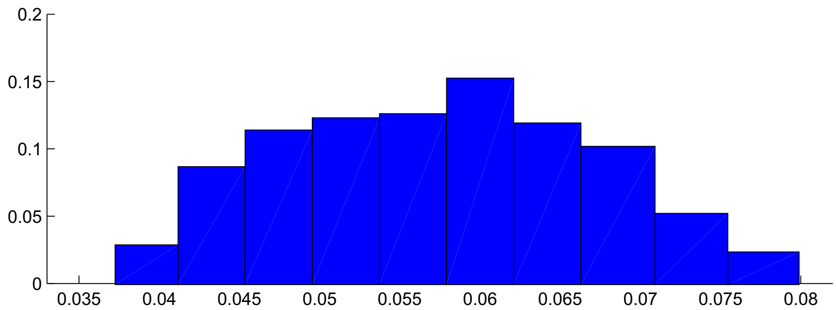

Fig. 6. Success probability distribution after the Monte Carlo simulation.

4.3. Third step: adjusting to a Beta probability distribution

In order to get a parametric probability distribution from the data simulated in the previous step, we decided to adjust the non-parametric distribution to a Beta probability distribution that models the *favorability* of the previous steps to an *a priori* probabilistic interval.

We state that this model is formally valid because the *favorability* interval $[u, l]$ is a uniform probability distribution that is a particular case of the Beta distribution with $\alpha = \beta = 1$ in Eq. (11) as shown by Rohatgi (1976).

The Beta probability distribution is defined as

$$B(\alpha, \beta) = \int_{0+}^{1-} t^{\alpha-1}(1 - t^{\beta-1})\mathrm{d}t \tag{11}$$

and the probability density function (pdf) is

$$f(x) = \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{B(\alpha,\beta)(b-a)^{\alpha+\beta-1}} \quad a \leqslant x \leqslant b;\ \alpha, \beta > 0 \tag{12}$$

where α and β are the shape parameters, a and b are the lower and upper bounds, respectively, of the distribution.

The α and β parameters are estimated from the result of the Monte Carlo simulation using the Maximum Likelihood Estimation method by solving the following set of equations

$$\psi(\hat{\alpha}) - \psi(\hat{\alpha} + \hat{\beta}) = \frac{1}{n}\sum_{i=1}^{n} \log\left(\frac{Y_i - a}{b - a}\right) \tag{13}$$

$$\psi(\hat{\beta}) - \psi(\hat{\alpha} + \hat{\beta}) = \frac{1}{n}\sum_{i=1}^{n} \log\left(\frac{b - Y_i}{b - a}\right) \tag{14}$$

5. Incorporating historical data and direct hydrocarbon indicators

5.1. Historical data

Frequently oil exploration companies invest in perforation of wildcat wells that are supported by analogous geologic conceptions, either in petroliferous basin-scale, as in system or play scale. When this happens, it is usual to use the historical observed frequencies of success of pioneering wells as good estimate in the representation of the certainty of the success of a prospect that has similar conception. However, this classical (frequencist) statistics representation implies the assumption that all relevant information is contained in the past history of analogous drilled wells. Such assumption would only be valid when assessing the value of conceptual prospects or in the petroliferous play-scale, which is not this case, as we are evaluating the perforation of a pioneering well in a specific position in the prospect.

Conversely, to affirm that all relevant information is represented only in the subjectivity of the geologic argumentation that support the prospect, implies to assume that the observed frequencies of success does not have any value. Such assumption would only be valid when assessing a completely new geologic conception where no historical information is available.

These two previously described situations show the objective (frequencist or *a posteriori*) approach and the subjective (*a priori*) approach.

In RCSUEX we implemented a mathematical model that put together the objective perspective with the subjective one. da Silva (2000) suggests to use a Bayesian model, where Beta distribution is the conjugate prior of the Bernoulli distribution. The reason why one would consider using the Beta distribution as the prior is because the Beta distribution and the Bernoulli distribution form a conjugate pair, so that the posterior distribution is still a Beta. The considered model is supported by the following theorem (Groot, 1970):

Theorem 1. *Let $W_1, \ldots, W_n$ be observations from a Bernoulli distribution unknown success probability Θ. Also suppose that a Beta distribution with parameters α and β is prior distribution for the unknown parameter Θ with $\alpha > 0$ and $\beta > 0$. Then, the posterior distribution of Θ when $W_i = w_i (i = 1, \ldots, n)$ is still a Beta with parameters $(\alpha + \delta)$ and $(\beta + n - \delta)$, where $\delta = \sum_{i=1}^{n} w_i$.*

In our application:

- $w_1, \ldots, w_n$ correspond to the historical observations of n pioneering wells that supposedly would have been drilled on the basis of analogous geologic conceptions, with δ equal to the number of successes (discovered), representing the objective knowledge (frequencist approach) about certainty of discoveries; $w_i = 1$, in case of a successful discovery; $w_i = 0$, when we have a dry well (or a not economic discovery);

- the Θ parameter is the probability of a pioneer, to be drilled with analogous geologic conditions of already drilled wells, to discover an oil field, representing the knowledge about the certainty of the discovery;
- the prior Beta distribution of Θ represents the subjective (*a priori*) knowledge (geologic and fuzzy) about the certainty of the discovery; and
- the posterior Beta distribution of Θ represents the subjective knowledge compound with objective (*a posteriori*) knowledge, according with Theorem 1.

The greater and significant the historical data, the bigger the weight of objective information in posterior representation. Conversely, in new exploratory areas or when using new geologic

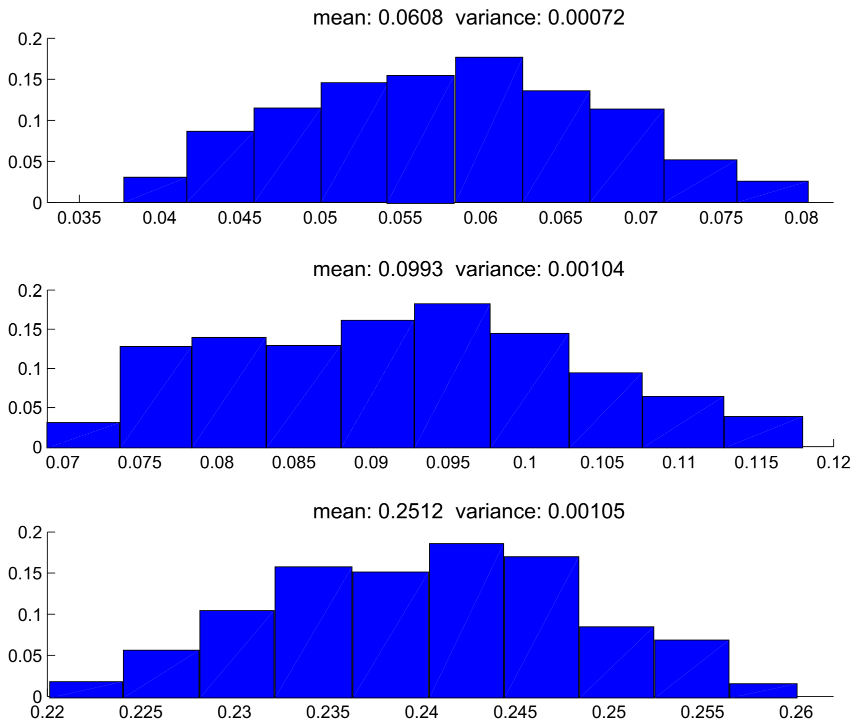

Fig. 7. Success probability distributions when incorporating historical observations.

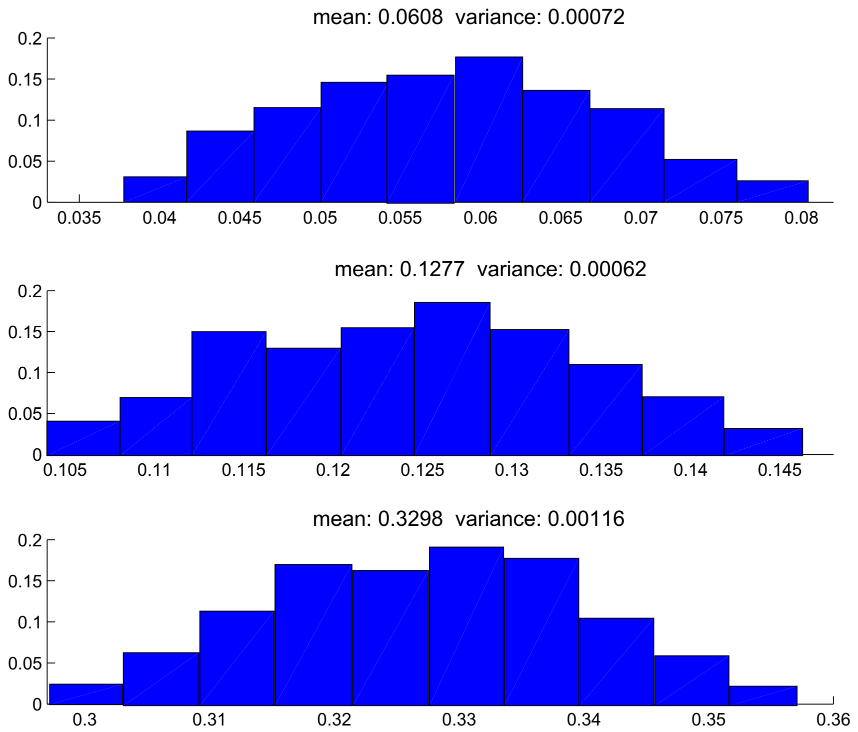

Fig. 8. Success probability distributions when incorporating DHI information.

models, with few historical data, greater will be the weight of prior (subjective) representation.

The following figure shows a brief example of this model implemented in RCSUEX. In Fig. 7a we can see the histogram of the success probability distribution for the prospect as result of subjective risk evaluation with fuzzy reasoning. Fig. 7b shows the posterior distribution obtained when the subjective model is conjugated with historical observations from 10 geologic analogous pioneering wells of which 4 was successful. In this case, we can observe that the average success rate grows up from 0.06 to 0.099. Fig. 7c shows the posterior distribution obtained when the same subjective model is conjugated with historical observations from 100 geologic analogous pioneering wells of which 40 was successful. In this case, we can observe that the average success rate grows up from 0.06 to 0.25, demonstrating the weight of the sample size in objective information.

5.2. Direct hydrocarbon indicators

Exploratory technologies and advances in seismic reflection imaging for direct identification of hydrocarbons, known as DHI – Direct Hydrocarbon Indicators, have becoming widely applied by petroleum industry as one of the most important elements in allowing companies to explore deepwater, since they often reduce geological risk to acceptable levels. They are based in geochemical principles (Belt & Rice, 2000) and mainly in geophysical analysis as the amplitude-versus-offset (AVO) (Rudolph, Fahmy, & Stober, 1998).

The same model described in previous subsection can be used to incorporate knowledge from DHI technologies as objective information to obtain the Beta posterior distribution of certainty of discovery. The weight of historical information is replaced by the weight of DHI information in the Bayesian model.

The weight of DHI is defined by the sequence of observations of Bernoulli samples $w_1, \ldots, w_n$ related with observation of n pioneering wells that hypothetically would be drilled based in DHI with the same intensity I and with δ occurrence of success. $w_i = 1$, in case of a successful discovery; $w_i = 0$, when we have a dry well (or a not economic discovery).

Here, I is defined as the linguistic expression that defines the geologic judgment about the DHI support for the prospect. In RCSUEX, we consider the following relationship between intensity I of DHI and the number of successful occurrence δ in the sequence of $n = 100$ hypothetical observations:

- $I =$ very weak $\Rightarrow \delta = 18$
- $I =$ weak $\Rightarrow \delta = 36$
- $I =$ moderate $\Rightarrow \delta = 54$
- $I =$ strong $\Rightarrow \delta = 72$
- $I =$ very strong $\Rightarrow \delta = 90$

The following figure shows a brief example of incorporation of DHI implemented in RCSUEX. In Fig. 8a we can see the histogram of the success probability distribution for the prospect as result

Table 1
Input variables, linguistic terms and system elements in the simple uncertainty model

Input variable	Linguistic terms	Geologic factor
Geothermal gradient	Low, good, high	Source rock
Coverage	Small, ideal, big	
Organic content	Poor, ideal, too much	
Maturity	Low, ideal, high	
Distance to a nearby producing well	Very near, near, middle, far, very far	
Net-to-gross ratio values	Low, moderate, high	Reservoir rock
Burial depth	Small, good, very good	
Control wells	Few, moderate, a lot	
Porosity	Poor, not-so-good, good, very good	
Reservoir continuity	Low, moderate, high	
Distance to a nearby producing well	Very near, near, middle, far, very far	
Effective thickness	Low, moderate, high	Seal rock
Predicted reservoir pressure	Low, moderate, high	
Seal continuity	Low, moderate, high	
Seismic control of seal	Yes, no	
Distance to a nearby producing well	Very near, near, middle, far, very far	
Path visibility	Low, good, very good	Migration
Pod distance	Near, moderate, far	
Distance to a nearby producing well	Very near, near, middle, far, very far	
Quality of seismic grid	Poor, good, very good	Trap
Kind of trap	Structural, stratigraphic, mix, hydrodynamic	
Control of velocity model	Poor, good, very good	
Sensitivity to depth conversion	Poor, good, very good	
Distance to a nearby producing well	Very near, near, middle, far, very far	
Geochemical modeling	Yes, no	Synchronicity
Confidence level in established lithology and age	Low, moderate, high	
Critical moment	Yes, no	
Hydrocarbon in similar age traps	Few, moderate, many	
Distance to a nearby producing well	Very near, near, middle, far, very far	

Table 2
Hypothetical prospect input variables and favorability evaluation

Input variable	Value	Evaluation
Geothermal gradient	150 °C	
Coverage	4300 m	Source rock
Organic content	10%	**Encouraging**
Maturity	450 °C	[57.54–93.77]
Distance to a nearby producing well	4.5 km	
Net-to-gross ratio values	85%	
Burial depth	3700 m	Reservoir rock
Control wells	3	**Encouraging**
Porosity	22	[56.58–93.29]
Reservoir continuity	0–100 level = 70	
Distance to a nearby producing well	4.5 km	
Effective thickness	100 m	
Predicted reservoir pressure	4300 psi	Seal rock
Seal continuity	0–100 level = 80	**Neutral**
Seismic control of seal	No	[39.43–79.43]
Distance to a nearby producing well	4.5 km	
Path visibility	0–100 level = 95	Migration
Pod distance	1.5 km	**Favorable**
Distance to a nearby producing well	4.5 km	[58.47–94.23]
Quality of seismic grid	200 m	
Kind of trap	Structural (confidence level = 100%)	Trap
Control of velocity model	0–100 level = 70	**Neutral**
Sensitivity to depth conversion	0–100 *level* = 50	[37.5–77.5]
Distance to a nearby producing well	4.5 km	
Geochemical modeling	Yes	
Confidence level in established lithology and age	0–100 level = 90	Synchronicity
Critical moment	Yes	**Encouraging**
Hydrocarbon in similar age traps	15	[57.05–93.52]
Distance to a nearby producing well	4.5 km	

of subjective risk evaluation with fuzzy reasoning. Fig. 8b shows the posterior distribution obtained when the subjective model is conjugated with a *very weak* DHI indication. In this case, we can observe that the average success rate grows up from 0.06 to 0.127. Fig. 8c shows the posterior distribution obtained when the same subjective model is conjugated with a *moderate* DHI indication. In this case, we can observe that the average success rate grows up from 0.06 to 0.329, demonstrating the weight of the DHI intensity in objective information.

6. An illustration of RCSUEX for a simple prospect risk assessment

6.1. Structure of RCSUEX

A risk analysis with RCSUEX is separated into three stages, namely:

(1) Defining variables, fuzzy sets and rules for each one of the six geologic factors and processes of the hydrocarbon system (source rock, reservoir rock, seal rock, trap, migration and synchronicity) according to the petroliferous model conceived by the domain experts;
(2) Evaluate the subjective probability perspective by assigning a value for each variable that composes the petroliferous model;
(3) Evaluate the posterior probability, by incorporation of historical data and DHI information.

These stages are selected through the menu bar in the user interface. The first stage is available only to users with privileged access rights, usually a team of company domain experts, that are responsible by generating the uncertainty petroliferous model that express geologic knowledge about the play system. Once the uncertainty model has been defined, explorationist users can select the petroliferous model and execute the other two stages, as well as visualize and print the systems variables, rules, outputs and reports.

6.2. The uncertainty petroliferous model

In this simple prospect example, a highly experienced geologist modeled the knowledge of a hypothetical play system. In order to accomplish this task, the modeler defined linguistic variables he believes are determinant in the reasoning process about the favorability of each element of the petroleum system. The linguistic variables are groups of fuzzy sets with partially overlapping membership functions. For each linguistic variable, the expert also defined linguistic terms. Linguistic terms are subjective categories for the linguistic variable. For example, for linguistic variable porosity, the domain set T (*porosity*) may be defined as follows (Tounsi, 2005):

$$T(\text{porosity}) = \{\text{very good}, \text{good}, \text{not so good}, \text{poor}\}$$

The linguistic input variables that supposedly influence each of system elements and their set of linguistic terms in its domain set are shown in Table 1.

The linguistic variables and fuzzy sets for each input variable was defined by the expert according with his knowledge about the most representative factors in petroleum evaluation. Note that some of these variables can be expressed by numeric values and are classified as interval variables (*e.g. Geothermal Gradient*). Other variables are just qualitative ordered variables and there is not a unit of measure to value them (*e.g. Path Visibility*). Some variables are 'crispy' in the sense that have a well defined value, as is the

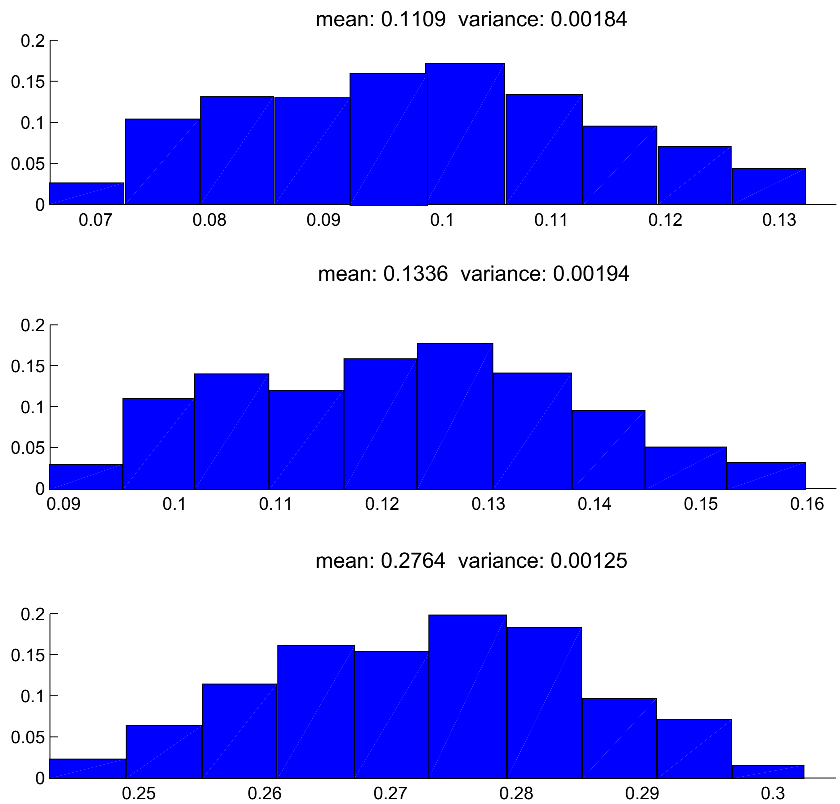

Fig. 9. Success probability distributions generated by RCSUEX for an hypothetical prospect with historical data and DHI information incorporation.

case of presence or absence of Geochemical Modeling when evaluating the Synchronicity Element.

Just after having defined the linguistic variables, the linguistic terms and its membership functions, the exploration expert defines a set of fuzzy inference rules for each geologic factor. In this example, he builded a preliminary set of 29 rules for the Source Rock Element, 31 rules for the Reservoir Rock, 21 rules for Seal and Trap, 13 rules for Migration and 18 rules for Synchronicity. In order to help tuning the system, each rule has a weight that consider its importance and coverage in determining the favorability of each element. Some examples of rules used in our system:

IF Organic Content IS Poor AND Distance to a nearby producing well IS Far THEN Source Rock IS Questionable ($W = 1.0$);

IF Geothermal Gradient IS Good AND Coverage IS Ideal AND Organic Content IS Ideal AND Distance to a nearby producing well IS Very near THEN Source Rock IS Favorable ($W = 1.0$);

IF Coverage IS Small THEN Source Rock IS Unfavorable ($W = 0.5$).

As previously shown, the output variable of each inference system is interpreted as favorability and is composed by the following linguistic expressions: "***favorable***", "***encouraging***", "***neutral***", "***questionable***" and "***unfavorable***", whose membership function has been presented in Fig. 4.

In Table 2 we show the values taken by the input variables when evaluating each Geologic Factor in a hypothetical prospect submitted to RCSUEX. The third column gives the favorability evaluation in terms of most adequate linguistic expression and uniform probability interval.

The chance factors of Geologic Factors of the prospect are combined using the Monte Carlo simulation technique in order to get the prior joint probability of success. In Fig. 9a we present the histogram of success probability distribution for the hypothetical prospect as a whole. In this case, we have a 11.09% prior Beta mean probability of success. Supposing that for this hypothetical prospect we have 6 pioneering wells in the same play area, 2 of each have discovered accumulations, Fig. 9b shows the posterior Beta probability distribution with average success rate of 13.36%. When incorporating DHI information to this evaluation, if we consider a weak DHI support for the prospect, with $\delta = 18$ for $n = 100$, we can observe in Fig. 9c that the average success rate grows up to 27.64%.

When this evaluation was presented to a team of experienced explorationists of the company, their opinion is that feel comfortable with the variables and the rules that express the qualitative knowledge about prospect appraisal and that the result obtained by the system was very near with their own judgment about the favorability of the hypothetical prospect.

7. Conclusions

In this paper, we have presented a new fuzzy-probabilistic representation of uncertain geological knowledge where the risk can be seen as a stochastic variable whose probability distribution counts on a codified geological argumentation. The risk of each geological factor is calculated as a fuzzy set through a fuzzy system and then associated with a probability interval. Then the risk of the whole prospect is calculated using simulation and fitted to a beta probability distribution. Finally, historical and direct hydrocarbon indicators data are incorporated in the model.

The hybrid fuzzy-probabilistic approach provides a strong modeling framework for a consistent and systematic utilization in argumentation of prospect appraisal. The fuzzy approach can deal with incomplete data and imprecise information typical in the exploratory domain. Observed frequencies of success coming from historical observations and direct hydrocarbon indicators are incorporated in the model.

The application of the theory of fuzzy sets to model the exploratory reasoning using linguistic terms allows to better understand the decisions and uncertainty concerned with the prediction of hydrocarbon accumulations. The process of definition of input variables and elaboration of rules permits knowledge and expertise aggregation by many company specialists and favors the treatment of more critical uncertainties.

We proposed that considering the fuzzy expert system output as favorability risk for each geological factor and associating it with probability intervals allows the connection between the fuzzy geologic interpretation and the probabilistic approach. With favorability of success now in the probabilistic domain, we showed how to use Bayesian probability theory in order to put together the objective perspective with the subjective one using the Bayesian model of Bernoulli distribution conjugated with Beta distribution, so it can be calibrated by comparisons with portfolio outcomes.

The system was applied in a simple hypothetical prospect example in order to evaluate the application of methodology. The proposed system is geologically sound and first results agreed with expected probability assessed by company experts. As chance is expressed numerically it can be directly used by corporate systems into economic analysis of exploration ventures. In the future we expect to apply the system on real petroleum prospects.

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