RC delay – 4: The Elmore delay - 3

Application of the Elmore delay formula to a (RC) wire.

Let $R$, $C$, and $l$ be the total line resistance, capacitance, and length.

\[ r = \frac{R}{l}; \quad c = \frac{C}{l}; \quad \Delta L = \frac{l}{N} \]

\[ V_{in} \quad r\Delta L \quad r\Delta L \quad V_{i-1} \quad r\Delta L \quad V_i \quad r\Delta L \quad V_{i+1} \quad r\Delta L \quad V_{out} \]

\[ \tau_{Dout} = \sum_{i=1}^{N} (ir\Delta L) c\Delta L = rc(\Delta L)^2 \left(1 + \frac{1}{2} + \ldots + \frac{1}{N}\right) = \]

\[ rc\left(\frac{l}{N}\right)^2 \frac{1+N}{2} = rcl^2 \left(\frac{1+N}{2N}\right) \]

\[ \tau_{Dout} = \lim_{N \to \infty} rcl^2 \frac{1+N}{2N} = \frac{rcl^2}{2} = \frac{RC}{2} \]

The delay of a wire is proportional to the square of its length.

Note: The Elmore formula applied to the RC lumped model gives $\tau_{Dout} = RC$

Source: Rabaey
Example 4.8 of Rabaey’s book: 10-cm-long, 1-μm-wide Al1 wire for which $r=0.075\ \Omega/\mu m$, $c=110\ \text{aF/μm}$.

\[
\tau_{Dout} = rcl^2 / 2 = 0.075\Omega/\mu m \cdot 110\text{aF/μm} \cdot (10^5\mu m)^2 / 2 = 41.3\ \text{ns}
\]

Note: The Elmore delay is, in general, not equal to the delay time. For a distributed RC network, the Elmore delay $\tau_D = 0.5\ RC$ whereas the delay time $t_d = 0.38\ RC$.
RC delay – 6

Example 4.8 of Rabaey’s book: 10-cm-long, 1-μm-wide Al1 wire for which $r = 0.075 \, \Omega/\mu\text{m}$, $c = 110 \, \text{aF}/\mu\text{m}$.

Distributed RC line
* this is DistributedRCline.cir file
v0 1 0 dc 0 pulse 0 1V 0 10ps 10ps 200ns 400ns
URC1 1 2 0 MURC L=100m
.model MURC URC rperl=75k cperl=110p
.end

SpiceOpus (c) 7 -> source DistributedRCline.cir
SpiceOpus (c) 8 -> tran 1ns 200ns
SpiceOpus (c) 9 -> setplot
   new
   New plot
Current tran2 Distributed RC line 1 (Transient Analysis)
SpiceOpus (c) 10 -> setplot tran2
SpiceOpus (c) 11 -> plot v(2) xlabel time ylabel Vout

![Plot of lumped and distributed RC delay](image)
RC delay – 7

Diffusion equation

\[ r c \frac{\partial^2 V}{\partial x^2} = \frac{2V}{a c^2} \]

\[ \tau(V_{out}) = \frac{r c L^2}{2} \]

Source: Rabaey
RC delay – 8

Step-response of RC wire as a function of time and space

Source: Rabaey
RC delay – 9

<table>
<thead>
<tr>
<th>Voltage range</th>
<th>Lumped RC network</th>
<th>Distributed RC network</th>
</tr>
</thead>
<tbody>
<tr>
<td>0→50% ($t_p$)</td>
<td>0.69 RC</td>
<td>0.38 RC</td>
</tr>
<tr>
<td>0→63% ($\tau$)</td>
<td>RC</td>
<td>0.5 RC</td>
</tr>
<tr>
<td>10→90% ($t_r$)</td>
<td>2.2 RC</td>
<td>0.9 RC</td>
</tr>
</tbody>
</table>

Source: Rabaey
When are the effects of the wire delay important?

Assume that the driver delay is $t_{pgate}$. The wire delay is

$$t_{pwire} = 0.38RC = 0.38r_w c_w L^2$$

The wire delay is important when $t_{pwire} \approx t_{pgate}$ or, equivalently

$$L_{crit} = \sqrt{\frac{t_{pgate}}{0.38r_w c_w}}$$
RC delay – 11

Example 4.8 of Rabaey’s book: 10-cm-long, 1-μm-wide Al1 wire for which \( r = 0.075 \, \Omega/\mu m \), \( c = 110 \, aF/\mu m \).

Distributed RC line 2
* this is DistributedRCline2.cir
* file
* the rise time is of the order of the
* RC time constant
v0 1 0 dc 0 pulse 0 1V 0 50ns 50ns +200ns 500ns
URC1 1 2 0 MURC L=100m
.model MURC URC K=2
+fmax=20G rperl=75k cperl=110p
.end

Response to pulse rise time=0
Response to pulse rise time=50 ns

Note that the internal resistance of the voltage source is zero in this example.

What if the rise time becomes much higher than RC?
Example 4.8 of Rabaey’s book: 10-cm-long, 1-μm-wide Al1 wire for which $r = 0.075 \Omega/\mu m$, $c = 110 \text{aF}/\mu m$.

What if the rise time becomes much higher than RC?
RC delay – 13

Source: Weste&Harris
Design Rules of Thumb

- rc delays should only be considered when $t_{pRC} \gg t_{pgate}$ of the driving gate
  
  $L_{crit} \gg \sqrt{\frac{t_{pgate}}{0.38rc}}$

- rc delays should only be considered when the rise (fall) time at the line input is smaller than RC, the rise (fall) time of the line

  $t_{rise} < RC$

- when not met, the change in the signal is slower than the propagation delay of the wire

Source: Rabaey
Inductance - 1

\[ \frac{d}{dt} L I \]

Inductive effects

- important for power grids (high current), clock networks (high speed), and wide busses (low resistance/unit length);
- may cause ringing/overshoot effects, reflection of signals, inductive coupling between lines (crosstalk), and switching noise in power lines

Clock trees and power/ground grid need to be designed carefully to avoid large clock skew, signal inductive coupling and ground bounce
Inductance - 2

- Inductance of a wire depends on its geometry and surrounding dielectric
- Extracting the inductance is in general a 3-D problem and is extremely time-consuming for complex geometries
- Inductance depends on the entire current loop; it is impractical to extract the inductance from a chip layout

Source: Rabaey, Weste & Harris
Inductance - 3

The Transmission Line

\[ \frac{2}{\alpha^2} \frac{\partial^2 V}{\partial x^2} = rC \frac{\partial^2 V}{\partial t^2} + lC \frac{\partial^2 V}{\partial t^2} \]

The Wave Equation

When \( r=0 \) \( \rightarrow \) signal travels at speed of light, which is smaller than speed of light in vacuum (300 mm/ns). In the real case, currents return in distant power lines and increase inductance thus reducing signal velocity.

When \( l=0 \) \( \rightarrow \) rc wire (diffusion equation)

Source: Rabaey
Inductance - 2

Source: Qi, CICC 2000

Fig. 10 Power and ground noise observed with RLC simulation.
Crosstalk is the coupling of energy from one line to another via:
- **Mutual capacitance (electric field)**
- **Mutual inductance (magnetic field)**

**Mutual Capacitance, \( C_m \)**

**Mutual Inductance, \( L_m \)**

Source: Intel