ECG Signal Processing Using Power Spectrum and Wavelet Transform: A Mathematical and Computing Aproach.

João Dovicchi*, João Bosco da Mota Alves*, Wemerson Délcio Parreira*, Li Shih Min†
*Remote Experimentation Laboratory
Departamento de Informática e Estatística
Universidade Federal de Santa Catarina (UFSC)
Florianópolis, SC, Brazil
Email: dovicchi@ieee.org, rexlab@rexlab.ufsc.br
†Departamento de Clínica Médica
Universidade Federal de Santa Catarina (UFSC)
Florianópolis, SC, Brazil
Email: li@rexlab.ufsc.br

***Abstract*— This paper presents the description of a formal mathematical and computing method of Discrete Wavelet Transform (DWT) combined with auto-correlation power spectrum. Detail segments (high-pass) of the first and second scale level of DWT ("lifting scheme") are convoluted using auto-correlation to achieve a more precise identification of cardiopathies which can be detected and represented for automated analysis due to power spectra differences. This study shows how ECG signals of normal and some cardiac pathologies can be compared to verify the applicability of the described method and improve its usability in ambulatorial clinic and other applicability for mobile monitoring.**

I. Introduction

Importance of biometric signal monitoring has increased recently for securing patient lives through detection of emergencies and abrupt changes in patient conditions. Some cardiac problems can occur during normal daily activities and may not be present or disappear when the patient is hospitalized. For this reason cardiac patients are particularly dependent on long-term monitoring equipment and on-line electrocardiogram (ECG) data transmission that can provide information for preventive diagnosis in advance.

There are several equipments commercialized for cardiac patient monitoring but its clinical use still lacks solution of technical issues. Rhee *et al.* [1] state that "long-term, ambulatory monitoring systems have not yet reached a technical level that is widely accepted by both clinicians and patients." Holter [2] has developed and tested an ambulatorial ECG monitoring. Although it has gained increasing popularity, it still has not precision and T-wave changes still do not have fidelity even after some new improvement, for it must be able to detect heart signals as a motion equipment and on an environment with various disturbances [3].

The main problem with Holters ECG signal assessment is the great amount of interference and the poor quality of the signal for a real time analysis. In this paper we present a new method for the study of ECG signals, using the Wavelet concept of multiresolution analysis (MRA) [4], [5].

A. Mathematical Background

MRA with a scaling function ϕ is a set of subspaces **V** in $\mathbf{L}^2[-\infty, +\infty]$:

$$\cdots V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \cdots$$

that has the following properties:

1) *Density* or $\cap_j V_j$ is dense in $\mathbf{L}^2[-\infty, +\infty]$.
2) *Separation* or $\cap_j V_j = 0$.
3) *Scaling* or $f(t) \in V_j \iff f(2t) \in V_{j+1}$ for all integers j and arbitrary functions f.
4) *Orthonormality* or funtions $\phi(t-k)$, for $k = 0, \pm 1, \pm 2, \ldots$ are orthonormal basis of V_0.

The simplest way to obtain a wavelet representation, where we can fix the level of time and of the frequency localizations, is to construct a function called "mother wavelet" such as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left(\frac{t-b}{a} \right), \tag{1}$$

where $a > 0$, $b \in \Re$ and ψ is a fixed function well localized in time and frequency domain [6].

The space of this function is a Hilbert space in $\mathbf{L}_2(\mathbf{R})$. The scalar product of two functions f and g in a Hilbert space is defined as $< f, g >= \int f(t)\overline{g(t)}dt$, so we can write a continuous wavelet transform of a function $f \in \mathbf{L}_2(\mathbf{R})$, as:

$$Wf(a,b) =< f, \psi_{a,b} >= \frac{1}{\sqrt{a}} \int f(t) \overline{\psi \left(\frac{t-b}{a} \right)} dt. \tag{2}$$

In equation 2, $\psi_{a,b}$ plays the same role as e^{iwt} in the definition of the Fourier transform [7].

The convolution of a set of sampled data with wavelet orthonormal basis coefficients can yield two sets of data since

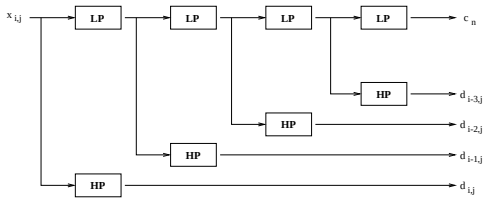

Fig. 1. The piramid algorithm of the forward Fast Wavelet Transform (FWT) and the repeated convolution of the sampled data with the wavelet coeffi cients Low-Pass (LP) and High-Pass (HP) fi lters.

they act as Finite Impulse Response (FIR) filters. Actually, wavelet coefficients can be used as low-pass and high-pass filters of complementary bandwidth. Like a FIR filter the recursive convolution can be writen as

$$y_k = \sum_{t=-\infty}^{\infty} c_{(k-t)} \cdot x_t$$

where y_k is the output of a set of inputs x_k convolved with $c_{(k-t)}$.

The new set of data can be splited in two sets: the low-pass and the high-pass segments. The low-pass segment can be used as a new input and can be convolved again. Notice that each time the process occurs, the low-pass set is decimated of a factor of 2 and the operation can be repeated until the sample interval is reached. So, if we want a 4 levels operation we will have to have 2^4 samples at least (see figure 1).

Since the high-pass set represents the detail coefficients for each level, the low-pass represents the data aproximation in that level, obviously with some loss of information. In order to analyze how much it changes in each level, comparison with other orthonormal basis can be done there is, each low-pass level can be transformed using another base to acquaint the frequency decimation.

The goal of this paper is to show that power spectrum of detail coefficients for each level can be used as an associate orthonormal basis to compare frequency spectra for each level. This procedure can differenciate normal ECG signals from ECG of cardiac pathologies, with more sensibility even when raw signals are used as input.

II. Methods

Data samples were downloaded from PhysioNet [8] at Universidade de S. Paulo (`http://physionet.incor.usp.br/physiobank/database/`) and were converted to data using Waveform Database (WFDB) software (`http://physionet.incor.usp.br/physiotools/wfdb.shtml`).

Sampled data were convolved with the scalling filters using "lifting method" coefficients (described below) as a quadrature mirror filter pair (complementary Low-Pass and High-Pass). The result yields two sets of data which are desampled by a factor of 2. Resulting sets of data are the high-pass filtered data (detail coefficients for this scale level) and the low-pass filtered data (aproximation coefficients at the scale level). The operation was repeated using the low-pass data as input for another level of transformation. Each level detail was used to obtain the power spectra where analysis were performed. The first 2 level high-pass samples were transformed using auto-correlation method to obtain the power spectra for frequency analysis and comparison. Through windowing methods, frames of the signal were used to compare normal and abnormal ECG signal data.

A. Lifting Method

The "lifting scheme" is a powerful method to design wavelet transforms first proposed by Win Sweldens [9]. Sweldens and Daubechies proved in [10] that this method allows to recover classical wavelet transform using a translation invariant discretisation of a plane.

The lifting scheme can be represented as in figure 2 as a chain of split/predict steps, where the even elements from one step become the input for the next step.

$$\begin{array}{cccccccccc}
x_0 & & x_0 & & m_0 & & m_0 & & m_0^{'} & \cdots \\
x_1 & & x_2 & & m_1 & & m_2 & & m_1^{'} & \cdots \\
x_2 & & x_4 & & m_2 & \rightarrow & m_1 & \rightarrow & d_0^{'} & \\
x_3 & & x_6 & & m_3 & & m_3 & & d_1^{'} & \\
x_4 & \rightarrow & x_1 & \rightarrow & d_0 & & & & & \\
x_5 & & x_3 & & d_1 & & & & & \\
x_6 & & x_5 & & d_2 & & & & & \\
x_7 & & x_7 & & d_3 & & & & &
\end{array}$$

Fig. 2. Split representation of the "lifting scheme". Means and differences are represented by letters "m" and "d" respectively.

One limitation of this method is the fact that it does not use any invariance propriety so it is not easy to get some convergence or some asymptotic algorithm of the cascade filter. This limitation can be supplied by auto-correlation, cross-correlation or a Fourier algorithm that can result in better information about the Wavelet function.

B. Auto-correlation and Power Spectrum

The convolution of two functions f and g (*cross-correlation*) correlates the signal f at some time i with a second function g at earlier times $i-j$. From the convolution $h(t)$ of two functions

$$h_i = f \circledast g = \int_{-\infty}^{\infty} f(i)g(i-j)dj \tag{3}$$

it can be stated the correlation function $Corr[f.g]$ where,

$$Corr[f,g] \equiv \int_{-\infty}^{\infty} f[u]g[t-u]du \tag{4}$$

Auto-correlation ρ_i of a periodic sequence $\{a\}_{i=0}^{i=N-1}$ is defined as:

$$\rho_i = \sum_{j=0}^{j=N-1} a_j \cdot a_{j+i} \tag{5}$$

For a continuous function $f(x)$, there is an auto-correlation function $Rf(x)$ defined as:

$$Rf(x) = \lim_{T \to \infty} \int_{-T}^{T} f(x) \cdot f(x+t)dt \qquad (6)$$

there is, the auto-correlation of a complex function is the cross-correlation of $f(x)$ with itself.

In order to supply the limitation of the "lifting scheme" and generate the power spectrum of the windowed data, an auto-correlation method was choosen. It was taken 50% overlaped windows of 1024 samples from all data signal and convoluted by auto-correlation to yield a power spectrum.

C. Computational Methods

Auto-correlation power spectra were calculated using correlation method (cspect) using SciLab 3.0 (Consortium Scilab, INRIA, ENPC, see `http://scilabsoft.inria.fr/`), runing on FreeBSD. Shell scripts to carry on automated tasks and Scilab programs were developed in our lab and are not documented yet, but can be solicited to authors by email.

III. Results

This section presents some results samples yielded by the signals analised using the method described previously. Authors have tested more than 53 archives of normal and pathological ECG signals. Each of them were windowed, overlaped and transformed, and presented very similar results. Some ECG signals presents visible changes for abnormal conditions, but in other cases do not. In those last cases power spectra of disease signals are similar to normal spectrum. Some cases of a slight supraventricular arrhythmia (SVA) can exemplify this. Its power spectrum can be analogous to normal conditions and can not be distinguished between the biological variation.

Figure 3 shows power spectra (auto-correlation) of ECG signal of normal (3(a)), supraventricular arrhythmia (3(b)) and malignant ventricular arrhythmia (3(c)).

Figure 4 shows power spectra of first level high-pass for the "lifting" wavelet of normal (4(a)), supraventricular arrhythmia (4(b)) and malignant ventricular arrhythmia (4(c)).

Figure 5 shows power spectra of second level high-pass for the "lifting" wavelet of normal (5(a)), supraventricular arrhythmia (5(b)) and malignant ventricular arrhythmia (5(c)).

Figure 6 shows a comparison of normal (6(a)), SVA (6(b)) and MVA (6(c)) spectra, where each scale shows frequency details. Power spectrum of the ECG signal is represented in gray. The two scale levels are represented in black; continuous for the first level and dash-lined for the second.

These samples represent only one window of the signal on the scale of interest (first and second levels of the Wavelet Transform), and other tests yielded similar results.

A. Discussion

This study has compared ECG signals of normal and some cardic pathologies. Since these experiments try to verify the applicability of the described computational method and its formal mathematical description, there is no improved analysis of results at pathological point of view.

Some authors [11] describe a normalized-value method that may be promising in classifying arrhythmias using a variability index that allows data to be assessed for any moment or during long time intervals. Laguna [12], [13] presented a threshold based detector of waveform limits, which locates each heart beat using a differentiated and low-pass filtered ECG signal as input.

There are many studies in this area, specially on Holter's ECG data because it is not precise and difficult to recognize some slight changes related to normal biometric samples. But there are still few results that can improve its usability in ambulatorial clinic and other applicability to mobile monitoring through wireless networks.

In our research we found some cases where normal biological variation can not be distinguished from abnormal case, specially when it is not so emphasized. Although it can be detected by a professional accurate examination, automated analysis can not be achieved with precision.

IV. Conclusion

This paper shows that auto-correlation of high-pass segment of transformed ECG signals using Wavelets "lifting method" can evidence parts of interest of these signals which can be used in future research for applicability to wireless monitoring of cardiac patients.

Authors believe that Wavelet transformed signal can help to improve usability for clinical diagnosis and applicability in wireless monitoring, even so it is necessary more research in this field.

Choosing the finest resolution is the next step. The finest resolution must be determined based on the smallest scale on which any effects of interest can be detected and represented. Only after that it will be possible to apply it to new information technologies (*e.g.* wireless monitoring).

This method can be applied to other biometrical signals where differences from normal conditions are difficult to find. From clinical point of view, this kind of work can help to guarantee safe and secure patients life.

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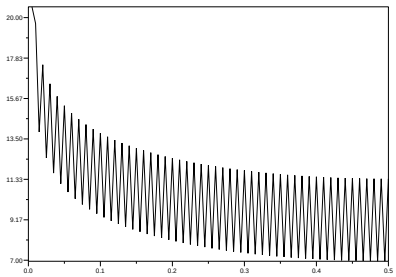
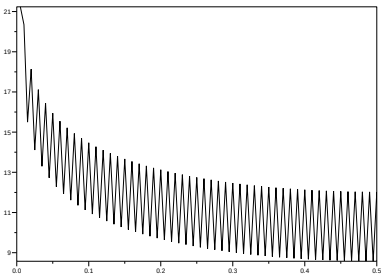
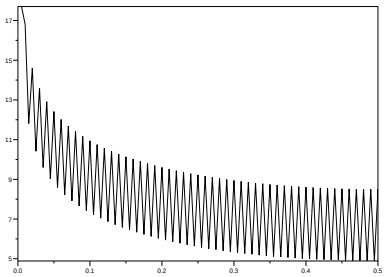


(a) Normal

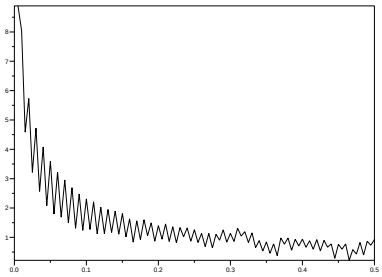


(b) SVA

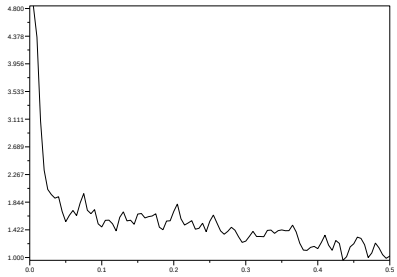


(c) MVA

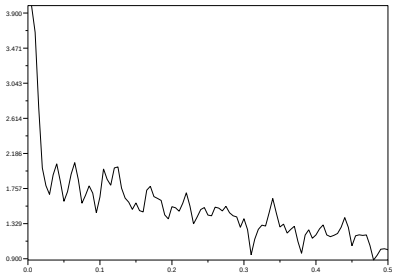
Fig. 3. Power spectra from ECG signals: (a) normal ECG, (b) supraventricular arrhythmia and (c) malignant ventricular arrhytmia.



(a) Normal

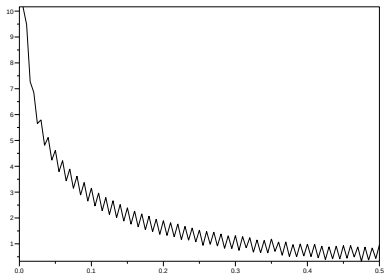


(b) SVA

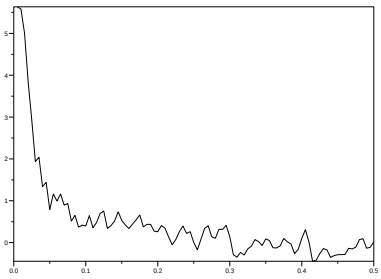


(c) MVA

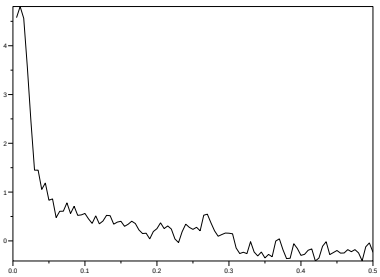
Fig. 4. Power spectra from high-pass segment of fi rst level Wavelet Transform: (a) normal ECG, (b) supraventricular arrhythmia and (c) malignant ventricular arrhythmia.



(a) Normal

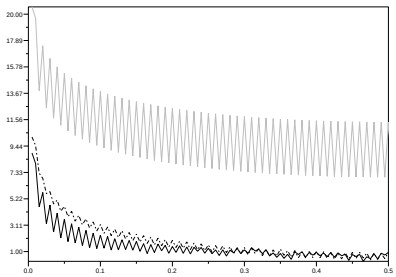


(b) SVA

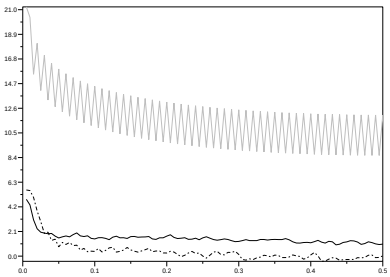


(c) MVA

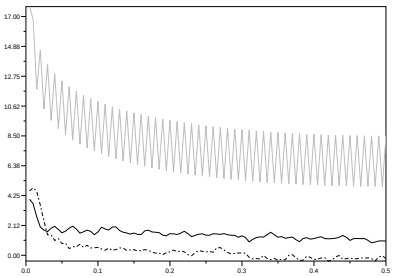
Fig. 5. Power spectra from high-pass segment of second level Wavelet Transform: (a) normal ECG, (b) supraventricular arrhythmia and (c) malignant ventricular arrhythmia.



(a) Normal



(b) SVA



(c) MVA

Fig. 6. Power spectra compared: (a) normal ECG, (b) supraventricular arrhythmia and (c) malignant ventricular arrhythmia.

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