SECTION 6: LOGIC, LANGUAGE, AND COGNITION

THE LOGIC OF AMBIGUOUS REFERENCE

Arthur Buchsbaum

Department of Informatics, Federal University of Santa Catarina, Brazil

Email: arthur@inf.ufsc.br

There are almost an infinite number of situations in mathematics, logic and everyday speech in which we have more than one object satisfying a given property, and we would like to use a name to denote an arbitrary object of this class.

So, in mathematics, for example, we denote a primitive of the function defined by $f(\boldsymbol{x}) = 2\boldsymbol{x}$ by "$\int 2\boldsymbol{x}d\boldsymbol{x}$", although we know that there exists more than one primitive for this function.

In syntax of formal logic, we usually define the expression "$\exists!\boldsymbol{x}\ \mathbf{A}$", whereon $\mathbf{A}$ is a formula, by "$\exists\boldsymbol{x}\ \mathbf{A} \wedge \forall\boldsymbol{x}\forall\boldsymbol{y}\ (\mathbf{A} \wedge \mathbf{A}(\boldsymbol{x}|\boldsymbol{y}) \rightarrow \boldsymbol{x} = \boldsymbol{y})$", whereon $\boldsymbol{y}$ is the first variable distinct of $\boldsymbol{x}$ that is not free in $\mathbf{A}$. It would be more natural to consider the expression "$\exists!\boldsymbol{x}\ \mathbf{A}$" stand for any expression of the form "$\exists\boldsymbol{x}\ \mathbf{A} \wedge \forall\boldsymbol{x}\forall\boldsymbol{y}\ (\mathbf{A} \wedge \mathbf{A}(\boldsymbol{x}|\boldsymbol{y}) \rightarrow \boldsymbol{x} = \boldsymbol{y})$", whereon $\boldsymbol{y}$ is distinct of $\boldsymbol{x}$ and not free in $\mathbf{A}$, dropping out the restriction about the alphabetical position of $\boldsymbol{y}$.

In everyday speech any noun preceded by an indefinite article is an ambiguous reference for any object of the correspondent collection. For example, the expression "a flower" is an ambiguous reference for any specific flower. So, the expression "a flower is beautiful" means, in a possible sense, that any flower is beautiful.

Besides ambiguous descriptions, there is a kind of assertions saying that a given object corresponds to some description.

In mathematics, by an abusive usage of the equality sign, we say that the function defined by $g(\boldsymbol{x}) = \boldsymbol{x}^2 + 3$ is a primitive of $f(\boldsymbol{x}) = 2\boldsymbol{x}$ by writing "$\int 2\boldsymbol{x}d\boldsymbol{x} = \boldsymbol{x}^2 + 3$".

In everyday speech, when we want to say that a rose is referenced by the description "a flower", we utter "a rose is a flower".

So, we isolated two key ideas concerning to ambiguous reference: *description* and *comprising*. We use two symbols for formalizing them: "ϒ" for description and "►" for comprising.

So, roughly speaking, according to our notation, we have:

- "$\int 2\boldsymbol{x}\mathrm{d}\boldsymbol{x}$" is a shorthand for "ϒ**g** (**g** is a primitive of the function $\mathbf{f}(\boldsymbol{x}) = 2\boldsymbol{x}$)";
- we can say that the function $\mathbf{g}(\boldsymbol{x}) = \boldsymbol{x}^2 + 3$ is a primitive of $\mathbf{f}(\boldsymbol{x}) = 2\boldsymbol{x}$ by writing "$\int 2\boldsymbol{x}\mathrm{d}\boldsymbol{x}$ ► **g**" or "$\int 2\boldsymbol{x}\mathrm{d}\boldsymbol{x}$ ► $(\boldsymbol{x}^2 + 3)\mathbf{d}\boldsymbol{x}$"; the reader should note the use of the sign "►" instead of the equality sign, as it is usually done, in a wrong way;
- we can also say "a rose is a flower" by the expression "ϒ***x*** (***x*** is a flower) ► ϒ***x*** (***x*** is a rose)".

A logic for dealing with these two ideas, enriching classical logic, is defined, from now on named "*Logic of Ambiguous Reference*", shortly **LAR**. We have defined a semantics and a sequent calculus for **LAR**, fitting to some basic intuitions. We also present some basic results concerning proof theory and semantics.

According to our intuition, such logic should take in account the following perspectives:

- LAR should be a conservative extension of classical logic;
- a description ϒ***x*** **P** should comprise, under reasonable restrictions, every term satisfying **P**, and only these terms;
- there should be a replacement rule for comprising, or, in a more formal way, "$\mathbf{P}(\boldsymbol{x}|\mathbf{t}),\ \mathbf{t} \blacktriangleright \mathbf{t'} \vdash \mathbf{P}(\boldsymbol{x}|\mathbf{t'})$", under reasonable restrictions, should be a rule of **LAR**.