A NEW GROUP OF LOGICS FEATURING NON CLASSICAL NEGATIONS

Arthur Buchsbaum
Departamento de Informática e de Estatística
Universidade Federal de Santa Catarina
Campus Universitário – Trindade
CEP 88040-900 – Florianópolis/SC – Brasil
Email: arthur@inf.ufsc.br

Tarcisio Pequeno
Laboratório de Inteligência Artificial
Universidade Federal do Ceará
Campus do PICI / Bloco 910
CEP 60455-760 – Fortaleza/CE – Brasil
Email: tarcisio@lia.ufc.br

Abstract

A new group of *paraconsistent* and *paracomplete* logics, and of logics combining the features of both, the so called *non alethic* logics, have been developed. Their conceptions follow from intuitions about two independent, although not at all disconnected, practical situations. One of them has ruled mainly the development of their syntax, while the other guided the formulation of semantics.

The first intuition is concerned with the problem of conducting reasoning under incomplete knowledge, where the need for conclusions to deal with practical situations requires the performance of inferences stronger than deduction, so of a non conservative, non tautological character. This is the kind of problem the famous *nonmonotonic logic*, developed as a branch of artificial intelligence, deal with. Typically, this kind of situation involves multiple evidences partially supporting conflicting conclusions and, quite frequently, there is not knowledge enough to enable a neat decision among them. Eventually, the reasoning under such circumstances must faces two natural alternatives: to assimilate all conflicting conclusions, treating them in the course of reasoning, or to promptly dismiss all of them. They have been named in the literature of artificial intelligence, respectively, as the *credulous* and the *skeptical* approach. In the first case, inconsistency arises among the admissible knowledge, leading to a *paraconsistent* treatment. In the second, *excluded middle* collapses, asking for a *paracomplete* treatment. A suitable combination of both alternatives, which is also a fair possibility, leads to a *non alethic* treatment. In very few words, this is how these calculi, to be here presented, have been motivated.

The second intuition, in which the semantic conception have been based, is concerned with a situation in which there are many subjects observing the same phenomenon under what is supposed to be the same circumstances. Nevertheless, the resulting observations may be not necessarily the same. In a real situation, this may come from the fact that the perfect equality of all observational conditions is really something never to be achieved. Those conditions are just *practically* the same, meaning the same as long as a selected set of relevant variables, the ones that according to the adopted theory can affect the final observation to a level within the measurement precision, is controlled. That much for a scientific experiment in the field of the so called *exact sciences*. In real life practical situations, or in a scenario concerning social science observation, for instance, or in the problem of *expert systems*, in the field of artificial intelligence again, an effective control of such a set of variables, or even a sharp demarcation of it, is out of question. Disagreement among observation is to be quite naturally expected. One has again to face two basic alternatives, as before: the assimilation of discordant *observation* to be reasoned upon, which brings about *paraconsistency*, or the dismissing of all the conflicting ones, which leads to uncovering of knowledge and to logical incompleteness, bringing about *paracompleteness*. A third possibility can be, again, a fair combination of both effects.

In our semantical framework, this plurality of views is expressed as a vector of (classical) individual valuations. This interpretation brings these logics quite close to some sort of *discursive logic*, the inconsistent tolerant logic originally conceived by *Jackowski* in his memorable pioneer work. According to a classification of paraconsistent systems attempted by *Routley* and *Priest*, these kind of logic should be characterized as *non adjunctive*, in their terminology. However, all the calculi here presented are strictly adjunctive – from **A** and **B**, $\mathbf{A} \wedge \mathbf{B}$ always follows. Amazingly, looking through the syntax, it can be seen that these calculi are in fact *positive plus* systems, according to the same classification. This is how the traditional da Costa's calculi of the $\mathbf{C_n}$ family, for instance, are classified. Odd enough, our logics hold properties that make then significantly distinct of the logics included in this group.

To begin with, all of them bear recursive semantics, undoubtedly, a remarkable feature for this kind of system. Besides, when compared with the correspondent da Costa's calculi $\mathbf{C_1}$, $\mathbf{P_1}$ and $\mathbf{N_1}$, under a suitable translation, these logics are strictly stronger than them. De

Morgan's and double negation law, for instance, are valid for all of them. On the other hand, several properties, valid to classical logic and certainly very welcomed in any logical systems, but that no longer hold for da Costa's systems, are here recovered. Among them, decidability by finite matrices, a form of substitution theorem and the existence of conjunctive and disjunctive normal forms. In fact, it has been suggested by Antonio Mario Sette, in a private conversation, that these calculi seems to be maximal approximations of classical logics, under reasonable constraints. Although this remark remains to be proved, the authors think it is likely to be true. At least it reflects their intent when designing those logics.

Correctness and completeness proofs have been provided for all logical systems presented. Through them, these two different approach converge, revealing that there are in fact very interesting correspondences between the two apparently independent views that have been taken here. In other words, these logics, with their respective completeness theorems, end up by revealing an underlying equivalence between these two different group of situations.

References

Buchsbaum, Arthur & Pequeno, Tarcisio, *Uma Família de Lógicas Paraconsistentes e/ou Paracompletas com Semânticas Recursivas*, Monografias em Ciência da Computação nº 5/91, Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro, 1991.

da Costa, Newton C. A., *On the Theory of Inconsistent Formal Systems*, Notre Dame Journal of Formal Logic 15, pp. 497-510, 1974.

da Costa, Newton C. A., *Logics that are both Paraconsistent and Paracomplete*, Rendiconti dell'Accademia Nazionale dei Linzei, vol. 83, pp. 29-32, 1989.

da Costa, Newton C. A. & Marconi, Diego, *A Note on Paracomplete Logic*, Rendiconti dell'Accademia Nazionale dei Linzei, vol. 80, pp. 504-509, 1986.

Reiter, Raymond, *Nonmonotonic Reasoning* , Annual Reviews of Computer Science, vol. 2, 1987, pp. 147-187.