THE LOGIC OF EPISTEMIC INCONSISTENCY

Tarcisio Pequeno Arthur Buchsbaum[1]
Departamento de Informática
Pontifícia Universidade Católica
Rua Marquês de São Vicente 225
22453-900 – Rio de Janeiro – Brasil

Abstract

The notion of epistemic inconsistency, referring to contradictory views about a same situation, is introduced. These contradictions reflect not an anomalous behavior of the state of affairs but the incompleteness (or vagueness) of our knowledge about it. The association of this phenomenon with nonmonotonic reasoning is discussed. A logic, with a calculus and a semantics, aiming to make precise this notion and to enable reasoning on these inconsistent views, without triviality, is presented.

1. INTRODUCTION

Deductive reasoning, our praised paradigm for correct, impeccable, contradiction free thinking, unfortunately has its application restricted to (ideal) situations where complete knowledge about the facts and their relations is available. For more realistic settings, the ones in daily life and in many A.I. applications, reasoning methods allowing the use of super deductive inference rules, such as reasoning by default, for instance, are required. Strictly speaking, these inference rules are not sound. It might happen some situation in which the premises are true but not the conclusion. As a consequence of this lack of soundness contradiction may eventually be achieved.

In [Pequeno 1990] it is argued that inconsistency is just a natural companion to nonmonotonic methods of reasoning and that paraconsistency (the property of a logic admitting non trivial inconsistent theories) should play a role in the formalization of these methods. The argument can be briefly stated as follows.

Nonmonotonic reasoning is applied to situations in which the knowledge is necessarily incomplete, eventually inaccurate as well, and very often involving information giving evidence to contradictory conclusions. Unlike deduction, this kind of reasoning cannot be performed on local basis, without appealing to context. In the course of reasoning the arguments interfere with each other, generating conflicts and promoting the defeat of partial conclusions.

Furthermore, there is no guarantee that every arising conflict can be resolved. It may perfectly happen that two opposite partial conclusions having equal rights to be achieved or, even if there is not such a perfect symmetry, it can happen anyway the available knowledge not enabling a clear decision in favor of one of the alternatives. Thus contradiction arises. In case of deduction this would carry out a revision of premises by the application of *reductio ad absurdum.* This is not the case here. There is no point in applying *reductio ad absurdum* to contradiction among defeasible conclusions.

In [Pequeno 1990] it is suggested that these contradictory conclusions should be assimilated in a single theory and reasoned out just as any other. This would emphasize the need for a better understanding of this kind of situations in order to provide a purely logical analysis for them. In other words, to achieve these contradictions that emerge in the course of reasoning is just to give the right account for the situation. Obviously this could not be done in classical logic. A special logic, a paraconsistent one, would be required.

---

[1] On leave from Universidade Federal do Ceará.

So, this is how nonmonotonic reasoning can lead to the adoption of paraconsistency. On the other hand, nonmonotonicity confers dynamics to paraconsistent reasoning. It allows incoming information eventually to remove contradictions and/or include new ones.

The contamination of nonmonotonic reasoning with irresolvable contradiction is a well known phenomenon. Examples that illustrate it, such as Nixon's diamond, have been recurrent in the literature on nonmonotonic logics. In spite of this recognition, there is an (understandable) resistance in assuming the inconsistent theories that these contradictions seem to imply. One approach to this problem has been simply to avoid the contradiction by dismissing both opposite conclusions. Another approach, taken by Reiter in his default logic, [Reiter 1980], has been to split contradictory conclusions into multiple extensions, each one internally consistent. We are more concerned here with the discussion of this second approach.

The splitting out of diverging default conclusions into multiple extensions has the effect of precluding the purely logical analysis of the whole situation. The contribution of extralogical mechanisms to deal with extensions, in order to perform reasoning, would be required. Furthermore, this approach has an undesired side effect which prevents default logic to avoid unintended extensions (and conclusions) in situations such as the famous "Yale shooting problem", discussed in [Hanks & McDermott 1987].

Consider the following example, taken from [Morris 1988]:

- Animals usually cannot fly;
- Winged animals are exception to this, they can fly;
- Birds are animals;
- Birds normally have wings.

This can be axiomatized, using Reiter's default, as follows:

- $\dfrac{\mathrm{an}(\boldsymbol{x}) : \neg\mathrm{fly}(\boldsymbol{x}) \wedge \neg\mathrm{wing}(\boldsymbol{x})}{\neg\mathrm{fly}(\boldsymbol{x})}$
- $\mathrm{wing}(\boldsymbol{x}) \rightarrow \mathrm{fly}(\boldsymbol{x})$
- $\mathrm{bird}(\boldsymbol{x}) \rightarrow \mathrm{an}(\boldsymbol{x})$
- $\dfrac{\mathrm{bird}(\boldsymbol{x}) : \mathrm{wing}(\boldsymbol{x})}{\mathrm{wing}(\boldsymbol{x})}$

The following reasoning will then be possible: given that Tweety is a bird, it follows from (3) that it is an animal, and from this and rule (1) that it cannot fly. By *modus tollens* on (2), if it cannot fly it is not winged. With Reiter's default logic this last conclusion prevents the application of the last default rule and therefore, from the single fact that the poor Tweety is a bird, it comes out this bizarre conclusion that it is wingless.

What happened so wrong here? The splitting into two extensions: one in which Tweety is winged and another in which it is not, didn't allow the reasoning to see that, by being a bird, therefore winged, Tweety constitutes an exception to rule (1), which makes this rule not being applicable. This information belonged to another extension and thus could not be seen from the unintended extension. Thus, the dissolving of conflicts by the splitting into extensions prevented the consideration of a relevant piece of evidence, causing the trouble of not handling properly the exception condition.

In [Pequeno 1990] it is presented a logic, the *Inconsistent Default Logic,* IDL in short, which benefits from a tolerant disposition towards contradiction, being able to solve this problem. A general IDL default rule reads as follows:

$$\frac{\mathrm{A} : \mathrm{B}\ ;\ \mathrm{C}}{\mathrm{B}?}$$

A is the *antecedent* of the rule and B its *default condition.* C is a *proviso* (its negation is an *exception condition* for the application of the rule). Finally, B? is the *consequent.*

This rule is a modification of Reiter's rule in accordance with the following considerations:

A defeasible conclusion can never have the same epistemic status as an irrefutable one, obtained from deduction. Thus in IDL the former is distinguished from the latter by the use of an interrogation mark

(?) suffixing defeasible formulas.

IDL implements the idea of accommodating conflicting views in a same extension. Therefore, in IDL the defeasible negation of a *default condition,* (¬B)? (we call it a *weak contradiction*), does not prevent the application of the default rule. In order to defeat a default application a *strong contradiction* ¬B is required. Nixon's example, for instance, when treated in IDL, generates just one extension, containing "pacifist(Nixon)?" and "¬pacifist(Nixon)?".

The *seminormal* part of a default rule is frequently used to express an exception condition. In IDL, C is really taken as a *proviso* for the application of the rule, receiving a differentiated treatment. In order to defeat the application of an IDL default rule by its proviso, a weak contradiction, (¬C)?, suffices.

We are now able to see how IDL works in Morris's example. The same argument could be constructed as before up to the temporary conclusion "¬winged(Tweety)?". But now this does not defeat the application of rule (4) and thus "winged(Tweety)?" is also achieved. This last conclusion, even being defeasible, is able to prevent the application of rule (1) (by means of its proviso). Therefore "¬fly(Tweety)?" is withdrawn together with "¬winged(Tweety)?". So, with IDL, only the expected conclusions that by being a bird Tweety are winged and can fly are obtained.

We leave to the reader to check it out that IDL would do equally well in the Yale shooting problem (lazy readers have the option to see it in [Pequeno 1990]).

We call epistemic inconsistency to inconsistencies in descriptions of a state of affairs reflecting not an inconsistency in the state of affairs itself but a lack in our knowledge about it. This term stands in opposition to ontological inconsistency, which refers to an inconsistent behavior of the reality itself (whatever this means) and has been used before, roughly with the same meaning as here, by [Rescher & Brandom 1980].

The Logic of Epistemic Inconsistency, LEI for short, has been designed aiming to make precise this notion. It is intended to reason out meaningfully contradictions resulting from reasoned out incomplete knowledge. Although to serve as the monotonic basis for IDL has been the main motivation for designing LEI, it has an independent existence and an interest in its own. The occurrence of inconsistencies arising from lack of knowledge is not restricted to nonmonotonic or inductive reasoning but, quite on the contrary, it is a very widespread phenomenon. It will be shown that some basic intuitions, reflected in the semantics of LEI, underlie many instances of it.

On the other hand, the ability to reason out contradictions without triviality characterizes LEI as a paraconsistent logic. Its properties can be studied in comparison with other members of this family. For instance, LEI is stronger than many other paraconsistent logics and admits a recursive semantics (in the sense of the meaning of an expression being determined by the meanings of its components). Such semantics, reflecting our intuitions on the notion of epistemic inconsistency, is given in the text. The calculus for LEI has been shown to be sound and complete with respect to it.

In the next sections the calculus for LEI will be presented followed by a discussion of the semantic intuitions assumed in its construction and by a precise formulation of its semantics.

2. THE CALCULUS OF EPISTEMIC INCONSISTENCY

The Calculus of Epistemic Inconsistency, CEI for short, is a paraconsistent calculus designed to cope with our intuitions about situations such as the ones described above. These intuitions are made precise in a semantics for the notion of epistemic inconsistency to be given later in this paper. Thus, this calculus is intended to reason out (meaningfully) the inconsistent theories arising on these situations. Its design aims to keep as many properties of classical logic as possible, without interfering with the properties required for the performance of this task.

Briefly stated, a paraconsistent logic is a logic in which triviality does not follow generally from contradiction. In its design CEI fulfills the requirements established in [Jaskowski 1948] and [da Costa 1974] concerning this kind of logical systems. As a matter of fact, the calculus CEI behaves classically for undoubting (monotonic, irrefutable) statements and paraconsistently for plausible (nonmonotonic, defeasible) ones. These two kinds of statements are distinguished in the language of the calculus by the use of an interrogation mark (?) suffixing the formulas of the last kind. When used in association with IDL, these marks are supplied by its default rules.

The following conventions are adopted in the presentation of CEI: the Greek letters α,β,γ denote arbitrary formulas in the language L' of the calculus, while the Roman capital letters A,B,C denote formulas in the language L without "?" (?-free formulas). "$\sim\alpha$" will be used as a short for "$\alpha \to p \wedge \neg p$", where p is an arbitrary sentential letter ("~" will be shown to behave as classical negation in CEI).

The axiomatics for CEI is the following:

(i) $\alpha \to \beta \to \alpha$;

(ii) $(\alpha \to \beta) \to (\alpha \to \beta \to \gamma) \to (\alpha \to \gamma)$;

(iii) $\dfrac{\alpha \quad \alpha \to \beta}{\beta}$;

(iv) $\alpha \wedge \beta \to \alpha$;

(v) $\alpha \wedge \beta \to \beta$;

(vi) $\alpha \to \beta \to \alpha \wedge \beta$;

(vii) $\alpha \to \alpha \vee \beta$;

(viii) $\beta \to \alpha \vee \beta$;

(ix) $(\alpha \to \gamma) \to (\beta \to \gamma) \to (\alpha \vee \beta \to \gamma)$;

(x) $((\alpha \to \beta) \to \alpha) \to \alpha$;

(xi) $(\alpha \to B) \to (\alpha \to \neg B) \to \neg\alpha$;

(xii) $\neg(\alpha \to \beta) \leftrightarrow \alpha \wedge \neg\beta$;

(xiii) $\neg(\alpha \wedge \beta) \leftrightarrow \neg\alpha \vee \neg\beta$;

(xiv) $\neg(\alpha \vee \beta) \leftrightarrow \neg\alpha \wedge \neg\beta$;

(xv) $\neg\neg\alpha \leftrightarrow \alpha$;

(xvi) $(\alpha? \to \beta?)? \to (\alpha? \to \beta?)$;

(xvii) $(\alpha? \vee \beta?)? \to (\alpha? \vee \beta?)$;

(xviii) $(\neg\alpha)? \leftrightarrow \neg(\alpha?)$;

(xix) $\alpha?$;

(xx) $\alpha?? \to \alpha$;

(xxi) $\begin{array}{c} \alpha \to \beta \\ \hline\hline \alpha? \to \beta? \end{array}$;

(xxii) $\begin{array}{c} \alpha \\ \hline\hline \sim((\sim\alpha)?) \end{array}$.

In the postulates above (iii), (xxi) and (xxii) are inference rules. The rule (iii) also holds as an implication, that is, $\alpha \wedge (\alpha \to \beta) \to \beta$ is a theorem of CEI. The same is not true for the other two rules. That is why a single bar is used for (iii) and a doubled bar for the others. The same convention will be adopted for derived rules in CEI, in order to indicate whether the use of these rules affects eventual applications of the deduction theorem.

Special attention must be paid to the Roman letter B at axiom (xi). This is a key axiom for the attainment of the selective paraconsistency of CEI. It restricts the axiom of absurdity to undoubting formulas (formulas without "?").

For the construction of this calculus, some guidelines were assumed. For instance, the following schemes were definitely rejected:

– $\alpha? \to \alpha$ (or even $\dfrac{\alpha?}{\alpha}$),

– $\alpha? \to (\sim\alpha)? \to \beta$ (or $\dfrac{\alpha? \quad (\sim\alpha)?}{\beta?}$).

By the first schema, defeasible knowledge would lead to irrefutable knowledge and, by the second, defeasible inconsistency would lead to (defeasible) trivialization. The implicative forms of the rules (xxi) and (xxii), for instance, would carry out $\alpha? \rightarrow \alpha$, while the rule $\alpha?, (\alpha \rightarrow \beta)? / \beta?$, although it may seem reasonable, entails $\alpha?, (\sim\alpha)? / \beta?$ in CEI. So, the distribution of "?" over the implication is not allowed. The same happens for $\alpha?, \beta? / (\alpha \wedge \beta)?$. In fact, these two rules can be shown to be equivalent in CEI.

The rule (xxi), used in conjunction with modus ponens rule, (iii), enables the derivation of $\beta?$ from $\alpha \rightarrow \beta$ and $\alpha?$. This kind of reasoning is one of the main mechanisms for propagation of "?" along inferences. Recall the example on birds and animals in the previous section. From the first default rule and the fact that Tweety is an animal, it follows "¬fly(Tweety)?" (With IDL reasoning this would be defeated later but anyway the argument is illustrative on how rule (xxi) works.) The statement (2) in the example is instantiated as

$$\text{wing (Tweety)} \rightarrow \text{fly(Tweety)}$$

Taking the contrapositive and applying rule (xxi) we get

$$\neg\text{fly(Tweety)}? \rightarrow \neg\text{wing(Tweety)}?.$$

This allows the application of modus ponens to get "¬wing(Tweety)?". The rule (xxii) states that we can't simultaneously be sure about α and consider that its classical negation is plausible, that is, all theory based on CEI having as theorems α and $(\sim\alpha)?$ is trivial. A particular case of this rule states that an irrefutable statement A should defeat its plausible negation $(\neg A)?$. This is done in IDL by the machinery of its default rules. Suppose that to the original statements of the Nixon's diamond example it is added that "anyone who promotes a war is definitely not a pacifist", together with the information that Nixon promoted the Vietnam War. This would lead to a categorical "¬pacifist(Nixon)", and it should defeat "pacifist(Nixon)?", otherwise the set of believes would become a trivial theory.

Let's give a look in the properties of CEI. Some theorems are in order. The first result states that the calculus CEI is in fact a classical calculus when restricted to undoubting formulas.

Theorem 1: All classical theorems hold to formulas of the language L (without "?") in CEI.

A corollary of this theorem is that the negation "¬" behaves classically as regards formulas of L:

$\vdash (A \rightarrow B) \rightarrow (A \rightarrow \neg B) \rightarrow \neg A;$

$\vdash \neg\neg A \rightarrow A.$

Theorem 2: The defined symbol "~" has indeed the properties of classical negation in CEI:

$\vdash (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \sim\beta) \rightarrow \sim\alpha;$

$\vdash \sim\sim\alpha \rightarrow \alpha.$

Theorem 3: Ali elimination and introduction rules for $\wedge$ and $\vee$ hold in CEI.

The following two theorems give an idea on CEI reasoning by showing samples of classical theorems which are kept and other which no longer hold in CEI, besides examples that illustrate its behavior.

Theorem 4: Among others, the following schemas are theorems or rules of CEI:

- $\alpha \vee \neg\alpha;$
- $\sim\alpha \rightarrow \neg\alpha;$
- $\alpha \rightarrow \beta \leftrightarrow \sim\alpha \vee \beta;$
- $\alpha \rightarrow \beta \rightarrow \neg\alpha \vee \beta;$
- $(\alpha \vee \beta)? \leftrightarrow \alpha? \vee \beta?;$
- $(\neg\alpha)? \leftrightarrow \neg(\alpha?);$
- $(\alpha? \rightarrow \beta?) \rightarrow (\alpha \rightarrow \beta)?;$
- $(\alpha \rightarrow \beta?) \rightarrow (\alpha \rightarrow \beta)?;$
- $\alpha // (\alpha \rightarrow \beta)? \rightarrow \beta?;$
- $(\alpha \wedge \beta)? \rightarrow \alpha? \wedge \beta?;$
- $\alpha // \beta? \rightarrow ((\alpha \wedge \beta))?;$
- $\neg(\alpha \wedge \neg\alpha).$

Notice that the thesis $\neg(\alpha \wedge \neg\alpha)$, which express the *non-contradiction law* (although in terms of a weak negation), is a theorem in CEI. Amazingly this does not prevent CEI of being paraconsistent, in spite of the condition of $\neg(\alpha \wedge \neg\alpha)$ not being a theorem has been stated in [da Costa 1974] as one requirement a paraconsistent calculi should fulfill. As a matter of fact, it is not a necessary requirement for the attainment of paraconsistency and, therefore, to avoid this theorem is against the aim of keeping as much classical theorems as possible. Furthermore CEI is not the only paraconsistent calculus in which $\neg(\alpha \wedge \neg\alpha)$ holds. This also happens to many relevant systems and to all paraconsistent calculi in [Buchsbaum & Pequeno 1991].

Theorem 5: Among others, the following schemas are <u>not</u> theorems or rules of CEI:

- $(\alpha \to \beta) \to (\alpha \to \neg\beta) \to \neg\alpha$;
- $\neg\alpha \to \sim\alpha$;
- $\neg\alpha \vee \beta \to (\alpha \to \beta)$;
- $\neg\alpha, \alpha\vee\beta \,/\, \beta$;
- $\to \beta \,/\, \neg\beta \to \neg\alpha$;
- $\alpha?\ (\neg\alpha)? \,/\, \beta?$;
- $\alpha? \,/\, \alpha$;
- $\alpha?, (\alpha \to \beta)? \,//\, \beta?$;
- $(\alpha \to \beta)? \,/\, \alpha \to \beta?$;
- $\alpha? \wedge \beta? \,/\, (\alpha \wedge \beta)?$;
- $\beta? \,/\, \alpha \to (\alpha \wedge \beta)?$.

This last theorem is useful in showing arguments that are not allowed. For instance, suppose we have A? and $(A \to B)?$. From the last we have $(\neg A \vee B)?$ and then $(\neg A)? \vee B?$, or even $\neg(A?) \vee B?$; but from this we cannot have $A? \to B?$, and then, by applying modus ponens, B?. On the other hand, by considering directly A? and $\neg(A?) \vee B?$, also B? does not follow.

Another important remark is that CEI reasoning does not mix up different extensions, as it should be expected. In a sense it is not adjunctive as regarding defeasible formulas. From $\alpha?$ and $\beta?$ we don't have $(\alpha \wedge \beta)?$, although having $\alpha? \wedge \beta?$. This property has a very significative effect on the reasoning. Suppose the following example:

- $\dfrac{\text{fly(x)? : fast(x)}}{\text{fast(x)?}}$
- $\dfrac{\neg\text{fly(x)? : cautious(x)}}{\text{cautious(x)?}}$
- $\text{fast(x)} \wedge \text{cautious(x)} \to \text{perfect-traveler(x)}$

From this last statement it can be derived (by rule xxi)

$$(\text{fast(x)} \wedge \text{cautious(x)})? \to \text{perfect-traveler(x)?},$$

but it would not be fair from "fly(Tweety)?" and "¬fly(Tweety)?" to conclude that Tweety is a perfect traveler and this is in fact not allowed in CEI. In other words, we have that may be Tweety is fast and may be Tweety is cautious, but not that may be Tweety is fast and cautious.

Every thing has a price to be paid for. The precautions against mixing up defeasible conclusions have a drawback, which ultimately comes from our decision of not distinguishing or talking about extensions. There are many situations in which effectively we want to combine defeasible conclusions, which would be in a same extension.

For instance, suppose we get from a default that "has-hoof(Incitatus)?" and "quadruped(Incitatus)?", and that we also have

$$\text{has-hoof(x)} \wedge \text{quadruped(x)} \to \text{equine(x)}.$$

Thus, we would like to be able to conclude "equine(Incitatus)?", but unfortunately this is not allowed for the feature just discussed.

A way out of this trouble would be to anticipate these valid combinations and, to remain safe, combine them through the default rules from which they come from. For instance, besides

$$\frac{\alpha : \text{has-hoof(x)}}{\text{has-hoof(x)?}} \text{ and } \frac{\beta : \text{quadruped(x)}}{\text{quadruped(x)?}}$$

it would be added their combination:

$$\frac{\alpha \wedge \beta : \text{has-hoof(x)} \wedge \text{quadruped(x)}}{(\text{has-hoof(x)} \wedge \text{quadruped(x)})?}$$

Now, "equine(Incitatus)?" can be inferred. Certainly this is not very elegant, but it is anyway effective in solving this problem.

Aiming at the construction of a theory of definition for CEI, a new abbreviation, the strong implication, is introduced: $\alpha \Rightarrow \beta$ is short for $(\alpha \rightarrow \beta) \wedge (\neg\beta \rightarrow \neg\alpha)$, as well as its corresponding double implication: $\alpha \Leftrightarrow \beta$ is short for $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

A replacement theorem for CEI, which would enable the eliminability of defined terms, can then be proved.

Theorem 6: (replacement) Let α' be a formula obtained from α by substituting β' for some occurrences (not necessarily all) of β. Then $\Gamma \vdash \beta \Leftrightarrow \beta'$ entails $\Gamma \vdash \alpha \Leftrightarrow \alpha'$.

The concept of a formula being closed is introduced now. It refers to a formula having all its components under the scope of an interrogation mark (taking "~" as in the language). A theorem assures that additional "?" suffixing this kind of formulas are irrelevant.

We say that a formula is *closed* if it has one of the forms $\alpha?$, $\neg\beta$, $\sim\beta$ or $\beta \# \gamma$, where β and γ are closed formulas and $\# \in \{\rightarrow, \wedge, \vee\}$.

Theorem 7: If α is a closed formula, then $\vdash \alpha \Leftrightarrow \alpha?$.

This concept plays a role in a version of the deduction theorem which holds for CEI.

Theorem 8: (restricted deduction) If there is a deduction of β from Γ,α, then $\Gamma \vdash \alpha \rightarrow \beta$, unless α is not closed and the rules (xxi) or (xxii) are used after the first time α occurs justified by being a premise.

3. SEMANTICS

The basic intuition to be captured by the semantics of LEI is the truthfulness relative to multiple observations of a same phenomenon, taken under different conditions, when the information about these conditions (or even on how the observations can be affected by them) is not available. It might happen, for instance, an experiment where a variation of conditions out of control (some variation is always the case in any experiment) is enough to affect the experiment to a level detectable by the instruments.

We are facing again a situation of insufficient knowledge leading to disagreement. It parallels our initial motivation about multiple extensions generated by a default theory when lack of knowledge does not enable the control of the selection among alternatives equally plausible.

A semantic framework reflecting the above scenario can be constructed as follows. A valuation V for an atomic proposition p is composed by a non-empty collection C of classical valuations for p (each one may be thought as expressing the opinion of an observer about p). There are two extreme alternatives for the definition of the truth value of p – a credulous and a skeptical one. By the credulous alternative p is taken as true if, for some classical valuation v belonging to C, v(p) is true. This alternative leads to a paraconsistent semantics, which we call semantics of maximization. This is the kind of semantics adopted in LEI.

By the other alternative p would be true only if it is true for all v in C. This could lead to a paracomplete semantics (semantics of minimization). A paracomplete logic is a logic in which the thesis expressing the excluded middle principle $(\alpha \vee \neg\alpha)$ is not valid.

A combination of these two alternatives would lead to a *non-alethic semantics.* A non-alethic logic is a logic both paraconsistent and paracomplete. All these alternatives have been explored for the construction of semantic systems and their corresponding calculi in [Buchsbaum & Pequeno 1991].

The semantics for LEI is given by the definition of a valuation function V, in terms of auxiliary functions v_{max} and v_{min}, recursively defined for each classical valuation v belonging to C. Roughly speaking, this semantics corresponds to the adoption of the first alternative as regards defeasible formulas (formulas suffixed by "?") and the second alternative to undoubting (?-free) formulas. In other words, V(A?) is true if A is true for some classical valuation v in C (A is true for someone), while V(A) is true iff it is true for all classical valuations v in C (A is true for everyone). The latter intends to capture the idea that the set of (monotonic) theorems of Γ corresponds to the intersection of all possible extensions (completements) of Γ.

Definition: Let C be a non-empty collection of classical valuations. For each $v \in C$, let vmax and vmin be functions from L' to {0,1} and let V be a function from L' to {0,1} too, such as the following conditions are satisfied:

- $V(\alpha) = l$ iff for all $v \in C$, $v_{max}(\alpha) = 1$;
- $v_{max}(p) = v_{min}(p) = v(p)$;
- $v_{max}(\neg\alpha) = l$ iff $v_{min}(\alpha) = 0$;
- $v_{min}(\neg\alpha) = l$ iff $v_{max}(\alpha) = 0$;
- $v_{max}(\alpha?) = l$ iff for some $v' \in C$, $v'_{max}(\alpha) = 1$;
- $v_{min}(\alpha?) = l$ iff for all $v' \in C$, $v'_{min}(\alpha) = 1$;
- $v_{max}(\alpha \rightarrow \beta) = l$ iff $v_{max}(\alpha) = O$ or $v_{max}(\beta) = 1$;
- $v_{min}(\alpha \rightarrow \beta) = l$ iff $v_{max}(\alpha) = O$ or $v_{min}(\beta) = 1$;
- $v_{max}(\alpha \wedge \beta) = l$ iff $v_{max}(\alpha) = l$ and $v_{max}(\beta) = 1$;
- $v_{min}(\alpha \wedge \beta) = l$ iff $v_{min}(\alpha) = l$ and $v_{min}(\beta) = 1$;
- $v_{max}(\alpha \vee \beta) = l$ iff $v_{max}(\alpha) = l$ or $v_{max}(\beta) = 1$;
- $v_{min}(\alpha \vee \beta) = l$ iff $v_{min}(\alpha) = l$ or $v_{min}(\beta) = 1$.

For any formula α in the language of CEI, the truth value of α is given by $V(\alpha)$.

Notice that, although the second alternative has apparently been adopted in our semantics for classical formulas, this does not lead to paracompleteness here, because V is defined in a way to make $\alpha \vee \neg\alpha$ valid.

The next is the key theorem in this section, stating the soundness and the completeness of the calculus with respect to the given semantics.

Theorem 9: (soundness and completeness) $\Gamma \vdash \alpha$ iff $\Gamma \models \alpha$.

The following theorem assures that the interpretation for classical formulas (?-free formulas) given in LEI is in accordance with the classical propositional semantics. It states that, under reasonable conditions, the ?-free logical consequences of a set of formulas in LEI are the same as the classical logical consequences of the ?-free formulas in this set. The reasonable conditions refer to having in the set only ?-free formulas or formulas containing an "?" suffixing a ?-free formula (avoiding arbitrary mixing up of interrogation marks). These are the kind of formulas generated naturally by a default theory consisting of classical formulas and IDL default rules.

A collection of formulas of LEI is said *normal* if it is non trivial and all of its formulas have one of the forms A or A?.

Theorem 10: Let Γ' be a normal collection of formulas in L' (L + "?") and Γ the set of ?-free formulas in Γ'. Then

$$\Gamma' \models_{LEI} A \text{ iff } \Gamma \models A,$$

where "$\models$" stands for the classical propositional logical consequence.

4. CONCLUSIONS

Our aim in this paper ha been to demonstrate that the idea of assuming all conclusions supplied by reasoning on incomplete knowledge, in spite of expressing inconsistent views, can be taken seriously. We are able to devise two possible objections to be raised against this idea. The first is a conceptual objection based on the argument that contradiction is indicative of error occurrence and therefore the efforts should be directed towards the correction of these errors and not in propagate them. It could be added that an inconsistent description is a description of nothing. The second is a technical objection based on the belief that, in any reasonable logic, inconsistent theories are trivial (everything is a theorem). We think that the logic presented here is evidence enough to remove the later objection. A logic able to perform reasoning in the presence of contradiction, without trivializing, being at the same time strong enough to make this reasoning useful, is perfectly possible. We would like to further discuss the first objection.

Contradiction is effectively a test for error. If a contradiction is deduced from a set of premises, this implies the inconsistency of this set. Semantically this means that no model can exist in which all of them are true. Thus it is a positive indication that these premises make a bad description of a given piece of a world. A similar practice is adopted in the methodology of the exact sciences. Contradictions among conclusions of a scientific theory, or among previsions of a theory and empirical observations, demand a revision of the hypothesis of the theory. To stay free of contradiction is one of the main methodological prescriptions of standard scientific practice.

The situation is quite diverse when common sense or artificial intelligence applications are considered. Then the inaccuracy of the knowledge is recognized in advance, and so the occurrence of contradictions does not provide such strong indication. It may demand an effort to get more precise information, but this refinement cannot be done beyond the limits of the knowledge available at a given time. In spite of it, reasoning must be done and decisions taken.

This kind of situation reveals that the role of reasoning is not exactly to come up with conclusions to be assumed as true in situations satisfying the premises. This picture fits well deductive reasoning and it is as prevalent as a paradigm for thinking that it is often taken as a general expression for reasoning.

Actually, the role of reasoning is to perform an analysis of the epistemic relations within the knowledge. It is to compose and judge evidences, to resolve conflicts (when possible) and to come up with relations between evidences and possible conclusions. These conclusions, in opposition to deductive ones, can never be detached from the premises in support of them. As has been pointed out, situations that satisfy the premises may not satisfy the conclusions. So, this kind of conclusions, more than a statement about the state of affairs, is a statement about our knowledge on the state of affairs. A contradiction then means simply that there are evidences in support of A as well there are also evidences supporting ¬A. There is nothing so striking about it. After words it is not even a contradiction. What can be a little striking is to take these conclusions altogether as a set of believes. This is precisely what is done here, but with the precautions of distinguishing this kind of belief and of treating them with a suitable logic. That is why we make a point in insisting about marking defeasible conclusions.

Perhaps the main point in these objections could be summarized in the observation that taking decisions on basis of the assumption that contradictory statements are true sounds nonsense and might even be dangerous. Although being correct, the observation misses the point. First of all because this is more a statement about the pragmatics of the situation, about the use to be done with a piece of information, than an epistemic one. In the second place because, as it has been argued, defeasible conclusions are not really intended just to be assumed as true. We certainly agree that an inference of A? ∧ ¬A? may add extra warning in the consideration of this kind of conclusion and this is reflected in the way the given calculus treats it.

It is worthwhile at this point to make a clear statement about the position we defend. We are not affirming here (at least not yet) that this is the only correct way to perform reasoning on such situations. The position we assume is a bit weaker. We just advocate, providing technical support, that this is a possible and legal way to do the things. On top of that, there is also a feeling that there are effectively situations in which this is really the appropriate approach to be taken. Another important point made here is the idea of providing a purely logical analysis of the reasoning, at least up to the point nonmonotonicity is itself accepted as logical. Our idea about default reasoning is to have it entirely performed by default rules extending a monotonic basis. LEI is offered here as a candidate for such a basis, but other alternatives could be the case. For instance, a paracomplete logic could be adopted if the situation requires a more cautious disposition.

The exploration of this kind of alternative is one of the directions for future research that we devise. As a first step in this direction, the authors have developed a whole family of non classical calculi coping with variations on a common semantic framework. These calculi and a study of their properties are presented in [Buchsbaum & Pequeno 1991]. We are also working out first order extensions for LEI and for all those calculi. Another point to deserve attention is to provide a semantics for the whole logic, encompassing its nonmonotonic part.

Acknowledgements

The authors are thankful to Newton C. A. da Costa and Richard Sylvan (former R. Routley) for discussions and criticism on earlier versions of this paper, and to Armando Haeberer, André Buchsbaum and Gerson Zaverucha for their help with revision and editing.

References

Arthur Buchsbaum & Tarcisio Pequeno (1991). *Uma Família de Lógicas Paraconsistentes e/ou Paracompletas com Semânticas Recursivas*. Monografias em Ciência da Computação no. 5/91, Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

Newton C. A. da Costa (1974). *On the Theory of Inconsistent Formal Systems. Notre Dame Journal of Formal Logic*, no. 15.

S. Hanks & D. McDermott (1987). *Nonmonotonic Logic and Temporal Projection*. Artificial Intelligence, no. 33.

S. Jaskowski (1948). *Un Calcul des Propositions pour les Systemes Deductifs Contradictoires*. Studia Soc. Scien. Torunensis, section A, 1.

P. H. Morris (1988). *The Anomalous Extension Problem in Default Reasoning*. Artificial Intelligence, no. 35.

Tarcisio Pequeno (1990). *A Logic for Inconsistent Nonmonotonic Reasoning*. Technical Report 90/6, Department of Computing, Imperial College, London.

Raymond Reiter (1980). *A Logic for Default Reasoning*. Artificial Intelligence, no. 13.

N. Rescher & R. Brandon (1980). *The Logic of Inconsistency*. Blackwell, Oxford.